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Correction of a Flaw in the Equations of Precessional Motion of a Spinning Top, of the Larmor Frequency, and of the Landé Factor

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Abstract

The equation of the precessional motion of a spinning top and the equation of the Larmor frequency require an additional factor of 2. The longstanding errors were caused by the wrongful treatment of a variable in the numerator of a differential quotient as vanishing although its true value was zero. As a consequence, one finds that, when doing things correctly, the dimensionless Landé factor of the electron is reduced to unity (instead of 2 in magnitude). Thus, there is no gyromagnetic anomaly of the spinning electron. The disputed result of Einstein's and de Haas' famous experiment is thereby vindicated. The results of experiments with macroscopic gyroscopes confirm the longstanding error, even though the measured angular velocity seems to match that predicted by the erroneous equation. To assess things correctly, unavoidable nutations must be considered (accompanying any precession of macroscopic objects), which result in a significant reduction in angular velocity. Therefore, the measured angular velocity of macroscopic gyroscopes is expected to fall short of the correct equation yield for purely precessional motion.

Keywords: Spinning top, Larmor frequency, Landé factor , Einstein, De Haas, nutation

I. Introduction

In textbooks, the precession velocity of a spinning top is given by the equation [see for instance: W. Thomson (Lord Kelvin) / P.G. Tait (1879), Section 105, pp. 79/80; H. Crabtree (1909), Art. 34, Fig. 20, pp. 37/38; A. Sommerfeld / F. Klein (1910), Chapter IX, §1, Fig. 113, equation (1), p. 762/763; L. Graetz (1917), Chapter 2, Fig. 40, p. 47, Section 105, pp. 79, 80; J.L. Synge/ B.A. Griffith (1942), Section 14.3, p. Fig. 151, p. 428; P.A. Tipler/ G. Mosca (2008), Chapter 10, Fig. 10-23, equation 10-19, p. 340; R.W. Pohl (2017), Chapter 6.11, Fig. 6.35, pp. 118/119]:
(1)

$$\omega \;=\; \frac{T_0}{J_{\;spin}}$$

Omega is the angular velocity of precession around the vertical **z**-axis, $\mathbf{J}_{\mathrm{spin}}$ is the angular momentum of the top around the axis of rotational symmetry (spin), $\mathbf{T}_0$ is the magnitude of the horizontal torque acting on the top in a direction perpendicular to the spin when $\mathbf{J}_{\mathrm{spin}}$ forms a 90 $^\circ$ angle with the vertical **z**-axis, and when the torque **T** is at its maximum value. If the spinning top is a permanently magnetized bar (magnetized in the direction of its axis of rotational symmetry, and hinged at its center of mass at the origin of coordinates), and if that bar finds itself in an external, homogeneous magnetic field **B** that points in the direction of the vertical **z**-axis, the angular velocity of precession around the vertical **z**-axis is, in text books, given by:
(2)

$$\omega \;=\; \frac{T_0}{J_{\;spin}} \;=\; \frac{\mu B}{J_{\;spin}}$$

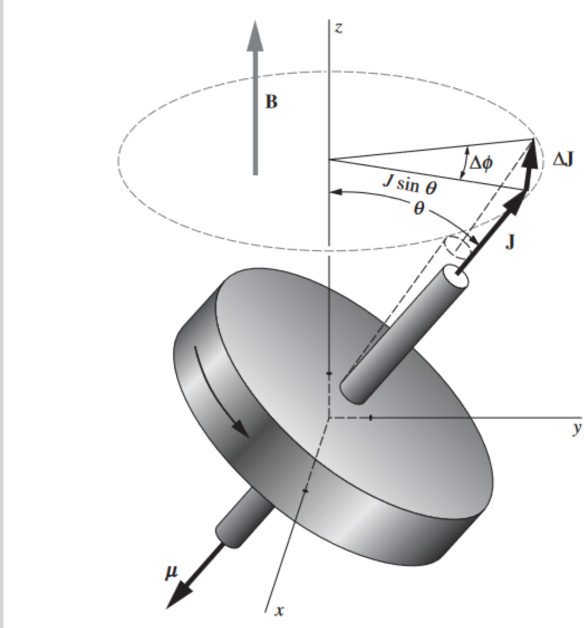
The parameter μ is the magnetic moment of the magnetized bar and **B** is the external magnetic field.

However, equations (1) and (2) are physically incorrect by a factor of two. This is demonstrated in the present study.

II. A first erroneous attempt of deriving the common equation of precessional motion

1) A spinning bar magnet (permanently magnetized in the direction of its axis of rotational symmetry, and hinged at its center of mass at the origin of coordinates) is assumed to perform a precession around the vertical **z**-axis. See the following figure [from Purcell/Morin (2013), Fig. J.1,
https://archive.org/details/ElectricityAndMagnetismPurcell3rdEdition]:

This precession is brought about by a torque **T** caused by a homogeneous magnetic **B**-field that points in the vertical **z**-direction. Thus, torque **T** has a



purely horizontal direction. In textbooks, (2) is then derived from reflections of the following kind [the example is taken from E.M. Purcell / D. J. Morin (2013), Appendix J (Magnetic Resonance), p. 821]:

"*In a short time Delta t, the torque adds to the angular momentum of our top a vector increment Delta J in the direction of the torque vector of magnitude μB sin theta Delta t*".

This statement is incorrect, simply because **Delta** $\mathbf{J}_{\text{horiz}}$ or ($\mathbf{dJ}_{\text{horiz}}$), that is, the magnitude of the generated angular momentum in a horizontal direction, is zero, and not just vanishingly small ("increment"). In other words, **Delta** $\mathbf{J}_{\text{horiz}}$ is not equal to μ**B sin theta Delta t**. Thus, $\mathbf{dJ}_{\text{horiz}}/\mathbf{dt}$, too, is zero. In greater detail: Although torque exists in a horizontal direction [see Purcell/Morin (2013), Fig. J.1 on page 822], it does not produce angular momentum in the direction of the torque once the spinning top precesses steadily around the vertical **z**-axis (which is presupposed to be the case in the situation depicted in Fig J.1). In other words: The angle between the internal axis of rotational symmetry of the spinning top and the vertical does not change, not even a bit. Moreover, the magnitude of the total angular momentum $\mathbf{J}_{\text{tot}}$ remains unchanged over time ($\mathbf{J}_{\text{tot\_2}}$=$\mathbf{J}_{\text{tot\_1}}$). Therefore, **Delta** $\mathbf{J}_{\text{horiz}}$ (or $\mathbf{dJ}_{\text{horiz}}$) is zero for the spinning top, and not vanishingly small (as textbook authors assume).

One should note that, when doing things correctly, the direction of the vector $\mathbf{dJ}_{\text{horiz}}$ is not expressed by reference to a stationary Cartesian system of coordinates; instead, the direction of the vector $\mathbf{dJ}_{\text{horiz}}$ is rotating with time, as does the torque $\mathbf{T}_{\text{horiz}}$. As a consequence, $\mathbf{J}_{\text{horiz}}$ is capable of undergoing changes in magnitude, but not in direction. Only then is it certain that any change in the vector $\mathbf{J}_{\text{horiz}}$ is caused by a torque, and not by something else. In other words, only then is it that we are allowed to postulate $\mathbf{T}_{\text{horiz}} = \mathbf{dJ}_{\text{horiz}}/\mathbf{dt}$.

There exists an analogy with the Lorentz force on a moving, charged particle. The Lorentz- force makes the particle move in a circle with radius $\mathbf{r}$, but it does not increase its kinetic energy $\mathbf{W}_{\text{kin}}$ or its linear momentum $\mathbf{P}$. This is despite the fact that a radial force $\mathbf{F}_{\text{r}}= \mathbf{dP}_{\text{r}}/\mathbf{dt}$ is constantly acting on the particle in a radial direction. The absence of an increase in kinetic energy can easily be explained by the fact that the two vectors $\mathbf{F}$ and $\mathbf{ds}$ are always at right angle with respect to each other, so that their dot product (yielding work $\mathbf{dW}$) is zero at any time. It's not so clear why the particle's momentum, too, remains invariant. Given that a radial force $\mathbf{F}_{\text{r}}$ is active all the time, the quotient $\mathbf{dP}_{\text{r}}/\mathbf{dt}$ should be different from zero, that is, as large as $\mathbf{F}_{\text{r}}$. Obviously, a radial counter-force is constantly acting on the particle. That counter-force is the "force" of inertia, or the centrifugal "force". The relevance of this counter force (centrifugal "force") is made obvious by the the fact that the radius of the circle performed by the charged particle in the magnetic field depends on a parameter that does not determine the magnitude of the *centripetal* force (=Lorentz force), but only that of the centrifugal "force"; that parameter is the mass $\mathbf{m}$ of the particle. The same is true for a gyroscope in stationary precessional motion. A horizontal torque generated by a magnetic or gravitational field is constantly acting on it, but this torque is neutralized by a counter-torque caused by the "force" of inertia. Thus, the net magnitude of $\mathbf{dJ}_{\text{horiz}}/\mathbf{dt}$ is zero in a stationary state of precessional motion.

If, instead, we define the vector $\mathbf{dJ}_{\text{horiz}}$ as a vector whose direction is defined by reference to a stationary Cartesian system of coordinates, it will be capable of undergoing changes in direction, and the quotient $\mathbf{dJ}_{\text{horiz}}/\mathbf{dt}$ will no longer be zero (since $\mathbf{dJ}_{\text{horiz}}$ will not be zero, but only vanishingly small). However, the so-understood quotient $\mathbf{dJ}_{\text{horiz}}/\mathbf{dt} = \mathbf{dJ}_{\text{spin}}/\mathbf{dt}$ is merely an expression of the angular velocity of precessional motion, which follows from the presupposed fact of a precession, and from nothing more. In other words: In that case, the equation $\mathbf{dJ}_{\text{horiz}}/\mathbf{dt} = \mathbf{dJ}_{\text{spin}}/\mathbf{dt}$ is correct *a priori* (and therefore doesn't qualify as a physical law, but as a tautology), given that a precession is occurring. But since it not sure whether or not $\mathbf{dJ}_{\text{horiz}}/\mathbf{dt}$ is also equal to $\mathbf{T}_{\text{horiz}}$ (which would be an empirical statement, that is, a statement that does not follow from the fact alone that a precession occurs), it is also not sure whether or not (1) and (2) are physically correct. For these two equations (and all "proofs") stand and fall with a substitution of $\mathbf{dJ}_{\text{horiz}}/\mathbf{dt}$ by $\mathbf{T}_{\text{horiz}}$, or of $\mathbf{dJ}_{\text{horiz}}$ by $\mathbf{T}_{\text{horiz}}\mathbf{dt}$, which is revealed when the train of thought that makes up the asserted proof is continued. Quod erat demonstrandum.

2) A more detailed way of advancing the same "proof" of (1) is the following:

– In a Cartesian **x,y,z**-diagram (where **z** is the vertical direction), the length of a vector whose origin coincides with the origin of coordinates represents the magnitude $\mathbf{J}_{\rm spin}$ of the angular momentum around the axis of rotational symmetry of the spinning top (bar magnet) at a given moment in time (for the distinction between $\mathbf{J}_{\rm spin}$ and $\mathbf{J}_{\rm tot}$ see below). The small (horizontal) vector $\mathbf{dJ}_{\rm horiz}$ mentioned by Morin/Purcell was added to the tip of vector $\mathbf{J}_{\rm spin}$ by these authors. The two vectors form a right angle with each other. The small angle **d phi** is the small angle between the positions of the precessing vector $\mathbf{J}_{\rm spin}$ at the beginning and end of the short temporal interval **dt**.

– As the angle **d phi** (expressed in rad) is equal to the quotient of the length $\mathbf{dJ}_{\rm horiz}$ of the short arc along the circumference of the tip's precession-circle and the radius $\mathbf{J}_{\rm spin}$ **sin theta** of the tip's precession-circle, Purcell/Morin claimed for this angle [**theta** is the angle between the vertical **z**-axis and $\mathbf{J}_{\rm spin}$]:

(3)

$$d\phi \;=\; \frac{dJ_{horiz}}{J_{spin}\sin\theta} \;=\; \frac{\mu B\ \sin\theta\ dt}{J_{spin}\sin\theta}$$

or

(4)

$$\frac{d\phi}{dt} \;=\; \omega \;=\; \frac{dJ_{horiz}}{J_{spin}\sin\theta\ dt} \;=\; \frac{\mu B\ \sin\theta\ dt}{J_{spin}\sin\theta\ dt} \;=\; \frac{\mu B}{J}\ spin$$

However, these equations are incorrect. The differential quotient $\mathbf{dJ}_{\rm horiz}\mathbf{/dt}$ appearing in (4) is zero (see above). Thus we have:

(5)

$$d\phi \;=\; \frac{dJ_{horiz}}{J_{spin}\sin\theta} \;=\; \frac{0}{J_{spin}\sin\theta} \;=\; 0 \;\neq\; \frac{\mu B\ \sin\theta\ dt}{J_{spin}\sin\theta}$$

or

(6)

$$\frac{d\phi}{dt} \;=\; \omega_0 \;=\; \frac{dJ_{horiz}}{J_{spin}\sin\theta\ dt} \;=\; \frac{0}{J_{spin}\sin\theta\ dt} \;=\; 0 \;\neq\; \frac{\mu B\ \sin\theta\ dt}{J_{spin}\sin\theta\ dt} \;=\; \frac{\mu B}{J}\ spin$$

The distinction between "zero" and "vanishingly small" (with respect to $\mathbf{dJ}_{\rm horiz}$ and "μ**B sin theta dt**") makes a difference of first order between the two sides of the inequation (6): In the left-hand half of inequation (6)], $\mathbf{dJ}_{\rm horiz}$ turns up in the numerator of a quotient $\mathbf{dJ}_{\rm horiz}\mathbf{/}$ **(**$\mathbf{J}_{\rm spin}$ **sin theta dt**), whose denominator **(**$\mathbf{J}_{\rm spin}$ **sin theta dt**) is vanishingly small. In other words, the quotient $\mathbf{dJ}_{\rm horiz}\mathbf{/}$ **(**$\mathbf{J}_{\rm spin}$ **sin theta dt**) is always zero, whereas the quotient μ**B sin theta dt/ (**$\mathbf{J}_{\rm spin}$ **sin theta dt**) = $\mu\mathbf{B/J}_{\rm spin}$ is a non-vanishing number.

III. A second erroneous attempt of deriving the common equation of precessional motion

A second (also unsuccessful) attempt to obtain (1) is the following:

The equation

(7)

$$\frac{d\vec{\mathbf{J}}_{spin}}{dt} = \vec{\omega}_p \times \vec{\mathbf{J}}_{spin}$$

was used as the starting point by some authors. **Omega** is the angular velocity of precession. $\mathbf{J}_{\mathrm{spin}}$ is the angular momentum of the top around the axis of rotational symmetry (spin).

Note that (7) presupposes that $\mathbf{J}_{\mathrm{spin}}$ undergoes a change in direction, but not in magnitude. If $\mathbf{J}_{\mathrm{spin}}$ undergoes a change in magnitude but not in direction, we have:

(7a)

$$\frac{\mathbf{d}\vec{\mathbf{J}}_{spin}}{dt} = \vec{\mathbf{T}}_{spin} \neq \vec{\omega} \times \vec{\mathbf{J}}_{spin} = 0$$

If $\mathbf{J}_{\mathrm{spin}}$ undergoes a change in direction but not in magnitude (as is the case when a gyroscope is precessing), we have:

(7b)

$$\frac{\mathbf{d}\vec{\mathbf{J}}_{spin}}{dt} = \vec{\omega} \times \vec{\mathbf{J}}_{spin} \neq \vec{\mathbf{T}}_{spin} = 0$$

As the next step, it is postulated that the right-hand side must be equal to the permanent torque $\mathbf{T}_{\mathrm{horiz}}$ applied to the gyroscope in a horizontal direction, or:

(8)

$$\frac{\mathbf{d}\vec{\mathbf{J}}_{spin}}{dt} = \vec{\omega} \times \vec{\mathbf{J}}_{spin} = \vec{\mathbf{T}}_{horiz}$$

The subscripts “spin” and “horiz” represent directions perpendicular to each other.

Next, (8) is converted into:

(9)

$$\left|\frac{\mathbf{d}\mathbf{J}_{spin}}{dt}\right| = |\vec{\omega} \times \vec{\mathbf{J}}_{spin}| = |\vec{\mathbf{T}}_{horiz}|$$

from which (1) is derived.

However, (8) and (9) are incorrect. The correct equations are:

(10)

$$\vec{\omega} \times \vec{\mathbf{J}}_{spin} = \frac{\mathbf{d}\vec{\mathbf{J}}_{spin}}{dt} = 2\vec{\mathbf{T}}_{horiz} \text{ and}$$

(11)

$$|\vec{\omega} \times \vec{\mathbf{J}}_{spin}| = |\frac{\mathbf{d}\vec{\mathbf{J}}_{spin}}{dt}| = |2\vec{\mathbf{T}}_{horiz}|$$

This is shown below. No reason was given by the authors why the factor in front of $\mathbf{T}_{\mathrm{horiz}}$ in (8) and (9) was simply unity, and not any other number.

One must realize that (7) is nothing but an expression of a given precessional motion, and does not assert anything other than that a precession is taking place. By itself, it does not imply a relationship with a torque $\mathbf{T}_{\mathrm{horiz}}$. With the same right, (7) could also be applied to the hand of a clock. Vector **J** in (7) can then represent the length of the hand, and vector **omega** can then represent the angular velocity of the hand. When knowing two of the three vectors that appear in (7), the third one is fixed, both in case of the clock and the gyroscope.

Therefore, when setting $\mathbf{dJ}_{\mathrm{spin}}/\mathbf{dt}$ equal to $\mathbf{T}_{\mathrm{horiz}}$ in (8) without any further arguing, one does not state an empirical law, but either *speculates* on an empirical relationship, or makes a *definition* of $\mathbf{T}_{\mathrm{horiz}}$. In the latter case, we have:

(11a)

$$\vec{\mathbf{X}} := \frac{\mathbf{d}\vec{\mathbf{J}}_{spin}}{dt} = \vec{\omega} \times \vec{\mathbf{J}}_{spin}$$

By itself, (11a) lacks of an empirical or logical necessity to set **X** equal to the torque $\mathbf{T}_{\mathrm{horiz}}$ (acting on the gyroscope), even though the dimension of the vector **X** is the same as that of a torque. Definitions are the results of arbitrary decisions, and they defy a rating of right or wrong. Even if we decide that **X** shall represent a torque, it will not be clear whether or not any factor should be placed in front of $\mathbf{T}_{\mathrm{horiz}}$. Whether nor not **X** should the same as $\mathbf{T}_{\mathrm{horiz}}$ with or without a factor in front of it, that is, whether or not a chosen definition is useful, is not revealed by (11a) proper. Instead, it is revealed by reflections that scrutinze how that decision would fit into the system of equations (laws) that already exist in physics, that is, whether or not the decision would lead to contradictions with those equations.

IV. The derivation of a correct equation of precessional motion

A correct equation is obtained by simply observing two basic principles: first, Newton's first law when applied to the vertical **z**-component of rotational motion (which is, different from the components of rotational motion in the **x**- and **y**-directions, not affected by the torque), and, second, the law of conservation of energy.

Let us imagine that the angular momentum of a spinning, permanently magnetized bar is oriented along the horizontal **x**-axis to start with. Gravity is absent. A homogeneous magnetic field **B** (not too strong) shall now be added which points strictly in the vertical **z**-direction. This situation was depicted in Purcell and D. J. Morin (2013), Fig. J.3., p. 823. In the following, $\mathbf{J}_{\text{spin}}$ is the magnitude of the angular momentum around the intrinsic axis of geometrical symmetry of the spinning bar which pointed in the horizontal **x**-direction before the external magnetic field **B** was switched on (with the latter pointing in the vertical **z**-direction), **T** is the torque caused by the homogeneous, external magnetic field **B** (which points in the vertical **z**-direction), the term $\mathbf{W}_{\text{kin-precession}}$ is the kinetic energy of the precessional motion, the term $\mathbf{J}_{\text{precession-z}}$ is the (vertical) **z**-component of the angular momentum of the precessional motion, $\mathbf{J}_{\text{total-z}}$ is the **z**-component of the total angular momentum, **theta** is the angle between the horizontal **x,y**-plane and the spinning bar (or the vector $\mathbf{J}_{\text{spin}}$) whose center sits at the origin of coordinates (note that **theta** is no longer the angle between the spinning bar and the vertical **z**-axis), μ is the (permanent) magnetic moment of the spinning bar, **omega** is the angular velocity of precessional motion, **M** is the moment of inertia of the precessional motion. Thus, we have the following two basic principles:

(12)

$$\int_{\theta=0}^{\theta=\theta} \sqrt{(T_x^2 \ + \ T_y^2)}\ d\theta \ = \ \int_{\theta=0}^{\theta=\theta} T_{horizontal}\ d\theta \ = \ W_{kin-precession} \ = \tfrac{1}{2}M\omega^2$$

or

(13)

$$\frac{\delta W_{kin-precession}}{\delta\omega} \ = \ M\omega \ = \tfrac{2}{\omega}\tfrac{1}{2}M\omega^2 \ = \tfrac{2}{\omega} \int_{\theta=0}^{\theta=\theta} \sqrt{(T_x^2 \ + \ T_y^2)}\ d\theta \ = \tfrac{2}{\omega} \int_{\theta=0}^{\theta=\theta} T_{horizontal}\ d\theta \ = \ J_{precession}$$

or

(14)

$$\omega \ = \ \frac{2\int_{\theta=0}^{\theta=\theta} \sqrt{(T_x^2 \ + \ T_y^2)}\ d\theta}{J_{precession}} = \frac{2\int_{\theta=0}^{\theta=\theta} T_{horizontal}\ d\theta}{J_{precession}} = \frac{2\ \mu B \int_{\theta=0}^{\theta=\theta} \cos\theta\ d\theta}{J_{precession}} = \frac{2\mu B \sin\theta}{J_{precession}}$$

and also

(15)

$$J_{total-z} = J_{spin-z} + J_{precession-z} = J_{spin}\sin\theta + J_{precession-z} = 0$$

or

(16)

$$J_{spin-z} = J_{spin}\sin\theta = -J_{precession-z}$$

Because the precession is only in the **z**-direction and in no other direction, we have the absolute magnitudes:

(17)

$$|J_{precession}| = |J_{precession-z}| = |J_{spin}\sin\theta|$$

Equation (14) thus converts into:

(18)

$$\omega = \frac{2\mu B}{J}{}_{spin}$$

The angular velocity of the precessional motion in the **z**-direction does not depend on **theta**. It is twice as large as reported by Purcell and other authors.

V. Energy and momentum balance of the process that brings about a steady precessional motion

As regards the torque around a horizontal axis (oriented always perpendicular to the axis of rotational symmetry of the bar, that is, to the axis of spin), the two basic principles mentioned above do not exclude a change in the magnitudes of the total angular momentum and the rotational energy as a result of the action of that torque. However, they set a limit on the extent of the change. That is, for the spinning top (bar) to precess around the vertical **z**-axis, it has to pick up energy to the extent of the kinetic energy of precessional motion. This energy is acquired before the final precessional motion is established. This is achieved by giving way to the horizontal torque.

In other words: it is incorrect to state that the horizontal torque does not result in an increase in the total rotational energy. Instead, the horizontal torque results in an increase in the total rotational energy and a change in the total angular momentum, but only to a limited extent.

For the stationary state of precession, we thus have (according to the Pythagorean theorem, and because $\mathbf{J}_{\mathrm{tot}}$ points in a strictly horizontal direction, given the net angular momentum of the gyroscope in the vertical direction is zero, while $\mathbf{J}_{\mathrm{precess}}$ points in a strictly vertical direction):

(19)

$$J_{tot}^2 = J_{spin}^2 - J_{precess}^2$$

or

(20)

$$J_{tot}^2 = M_{spin}^2\omega_{spin}^2 - M_{precess}^2\omega_{precess}^2 = M_{spin}^2\omega_{spin}^2 - \frac{4T^2 M_{precess}^2}{J_{spin}^2} = M_{spin}^2\omega_{spin}^2 - \frac{4\mu^2 B^2 M_{precess}^2}{J_{spin}^2}$$

One realizes that while the total energy of rotational motion increases, the total momentum declines as a result of the process that generates the precessional motion. Once that motion has been established, the horizontal torque is no longer capable of changing (=reducing) the magnitude of the total angular momentum $\mathbf{J}_{\text{tot}}$ any longer. If the magnitude of the total angular momentum could be changed any further, at least one of the two principles mentioned above would be violated.

The decrease in the total angular momentum of the gyroscope during the process (at the end of which the gyroscope precesses steadily) is accounted for by the principle of conservation of angular momentum. While the external magnetic field that was generated by a big magnet exerted a torque on the spinning bar magnet, the magnetic field of the spinning bar magnet exerted a counter-torque that acted on the big magnet. The counter-torque produced an angular momentum of the big magnet that hadn't been there before. Since the sum of all angular momenta must remain invariant, it follows that the total angular momentum of the gyroscope had to decline.

VI. The correct equation of Larmor precession

To obtain the equation of precession of a spinning top when the spinning top is not a permanently magnetized spinning bar, but a charged, spinning particle (Larmor precession), the quotient $\mu/\mathbf{J}_{\text{spin}}$ is simply replaced by $\mathbf{q/2m}$; where $\mathbf{q}$ is the amount of electric charge of the particle and $\mathbf{m}$ is its mass [the substitution of $\mu/\mathbf{J}$ by $\mathbf{q/2m}$ dates back to O.W. Richardson (1908)]. This substitution holds true (in classical physics) for all homogeneously charged objects that are rotationally symmetrical. Therefore, (18) can be replaced by:

(21)

$$\omega = \frac{2\mu B}{J_{spin}} = 2\,\frac{qB}{2m} = \frac{qB}{m} = g\,\frac{qB}{m}$$

The parameter $\mathbf{g}$ is a dimensionless correcting factor required when the spinning particle does not behave classically. If $\mathbf{g}>1$, the magnetic moment is greater than expected, or the angular momentum is smaller than expected (or both). If **g**<1, the magnetic moment is smaller than expected, or the angular momentum is greater than expected, or both. Experimental measurements the Larmor frequency (that yield **omega**) suggest that the Landé factor **g** is equal to 1, so that the particle behaves classically.

However, if the wrong equation (2) is taken as a starting point, one arrives at the wrong equation:
(22)

$$\omega \;=\; \frac{\mu B}{J_{spin}} \;=\; \frac{qB}{2m} \;=\; g\,\frac{qB}{2m}$$

Measurements of the Larmor frequency (that yield **omega**) then wrongly suggest that the Landé factor **g** is equal to 2 for the particle (electron), so that the particle does not behave classically.

VII. The correspondence of cyclotron and precessional motion as a cross-check of the equation of precessional motion

It is worth noting that the cyclotron angular velocity of charged particles in a magnetic field is identical to the right-hand side of (21) (if **g**=1) [Ch. Kittel (2005), Chapter 8, Equation 30, p. 200]. The equation of cyclotron motion can easily derived from the following two equations (the left-hand side of the first equation is an expression of the Lorentz force that acts on the charge **q**, the right-hand side of the first equation is an expression of the centrifugal force that acts on the inert mass **m** of the charge, **r** is the radius of a circle path):
(23)

$$qvB \;=\; \frac{mv^2}{r} \quad \wedge \quad v \;=\; \omega r \quad \Rightarrow \; \omega_{cyclo} \;=\; \frac{qB}{m}$$

From (23), we learn that the circular motion at that angular velocity in a homogenous magnetic field does not result in any internal pressure or traction (inflicted by a Lorentz-force) on the spinning, charged object. In other words: it results in no “tidal force”. This is because **omega** does not depend on **r**.

Furthermore, the cyclotron angular velocity equation (23) is valid for all circular paths (around the magnetic field lines) of electrically (volume-)charged bodies caused by the Lorentz force. In other words, given that the path is circular, it must obey equation (23).

With regard to a spinning, non-magnetic, but electrically (volume-)charged bar (hinged at its center of mass at the origin of coordinates) that was oriented along the horizontal **x**-axis before the external magnetic field turned up, all of its eventual circular motion of precession around the lines of the **B**-field (that is, around the vertical axis of a **x,y,z**-system of coordinates) can be regarded as cyclotron motion, given the existence of an external magnetic field and hence a Lorentz force.

Likewise, when replacing the spinning bar with a spinning macroscopic sphere (subject to an external magnetic field) homogenously filled with fixed electric charge in its interior, the precessional motion of this sphere must obey the equation of *cyclotron* angular velocity as well as the equation of *precessional* angular velocity. This requirement is met if (18) and (21) are accepted as the correct equations for the angular velocity of precessional motion. However, this requirement is *not* satisfied if (2) is adopted. This presents a cross-check of (18) and (21).

VIII. A vindication of the result g=1 obtained by Einstein and De Haas in their famous experiment

It is well known to every scholar that Einstein and de Haas performed an experiment aimed at a determining the quotient $\mu/\mathbf{J}$ for the electron [A. Einstein / W.J. de Haas (1915)] by direct mechanical measurement of both μ and $\mathbf{J}$ in a sample material (i.e., without resorting to the Larmor frequency). The outcome was a confirmation of the classical equation $\mu/\mathbf{J} = \mathbf{q/2m}$, and the factor **g** was thus found to be equal to unity [see P. Galison (1982), p. 297: "*Einstein's theoretical prediction corresponded to a g-factor of 1; his and de Haas' empirical result was equivalent to a g-factor of 1.02 with an error of 0.10*."].

Since 1915, the experiment has been repeated by others [for instance by S.J. Barnett/ L.J.H. Barnett (1925)] with varying, though not compelling results. P. Galison (1982) provided an overwiew.

It was because of the incorrect equation (22) of the Larmor precession that the scientific community has eventually tended towards a value of two (rather than one) for the Landé factor **g**. It was observed that matter was sending out electromagnetic waves the origin of which was seen in the precessional motions of particles. To describe these precessional motions, the (wrong) Larmor equation (22) was used. Therefore, based on the observed radiation frequencies or the observed **omega** (and knowing **B**, **q** and **m**), the conviction prevailed that **g** had a value of (approximately) two. The experimenters S. J. Barnett/L.J.H. Barnett (1925, p. 128) expressed this development as follows:

"*Our phenomenon is undoubtedly connected closely with the Zeeman effect, as our magnetons may be considered to be executing regular precession upon them brought about by the rotation. ... As Landé has suggested, the anomaly of the Zeeman effect .... is probably related to the anomaly in our phenomenon. This anomaly Landé and Sommerfeld have attempted to explain by a process which appears to be ... attributing to this a value of [g] equal to m/e [g=2]... .*"

However, with the correct equation (21) this argument becomes baseless, and **g** is found to be equal to unity (as had been asserted by Einstein and De Haas).

IX. Laboratory experiments with macroscopic gyroscopes

Laboratory experiments aimed at determining the precession rates of gyroscopes are regularly performed for pedagocical purposes in undergraduate courses at colleges and universities. As a result of these experiments, incorrect equation (1) appears to be confirmed.

There is a simple explanation for this outcome; in these cases, precession is necessarily accompanied by nutation. This reduces the angular velocity of the precession by approximately half.

In greater detail, when the spinning wheel of a demonstration gyroscope is released from rest, it drops slightly (see above). Thereby, it picks up some kinetic energy. After reaching the equilibrium angle **theta**, the kinetic energy of the vertical fall is not instantly converted into the kinetic energy of horizontal motion along a perfectly circular path of precession. Instead, the fall of the gyroscope overshoots the equilibrium angle **theta** to some extent. As a consequence, the path of precession of any point of the figure axis (axis of rotational symmetry) displays twists and turns, and forms a succession of numerous "U"s that sit side by side [see J. Hanks (1994), Fig. 4.2 A, p. 18]. The total length of the path is approximately doubled. However, the resulting speed along this twisted path is determined (and limited) by the amount of potential energy converted into kinetic energy during the vertical fall. Hence, the kinetic energy is not larger than that in the case of a perfect, that is, an undisturbed precession circle.

Given that the total length of the circular path is enlarged (roughly doubled) by the twists and turns, the time needed to complete a full precession circle has thus increased (roughly doubled), and the angular velocity is only approximately half of that without nutation.

To put it the other way round: If (1) were correct, the experimental results would have to fall significantly short of what is predicted in (1), given that the total path, due to nutation, is much longer than 2 **pi r**, and also given that the velocity along the path cannot be higher than it would be without the nutation.

When it comes to electrons and their precession, there is no need to expect a nutation.

Finally, one should be aware of the following consequence: In case empirical results lead to a discarding of Equation (2), at least one of its starting points, that is, the principle of energy conservation and the principle of conservation of angular momentum, would thereby be proved to be empirically wrong.

X. Results

By using Newtonian mechanics and the principle of conservation of energy, it was shown that the equation of the precessional motion of a spinning top and the equation of the Larmor frequency require an additional factor of two. The longstanding error in the literature was caused by the wrongful treatment of a variable ($\mathbf{dJ}_{\mathrm{horiz}}$) in the numerator of a differential quotient as vanishing although its true value was zero. When defining the vector $\mathbf{dJ}_{\mathrm{horiz}}$ to be capable not only of changes in magnitude, but also in direction, it is no longer zero (during the stationary state of precession), but only vanishingly small (which makes an important difference). But then $\mathbf{dJ}_{\mathrm{horiz}}/\mathbf{dt}$ is no longer a correct expression of the torque that acts on the gyroscope.

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