A simple derivation of Einstein’s field equation, based on the extended relativity principle only, and the stunning consequences for Kaluza’s theory

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By Andreas Trupp

Abstract: General Relativity has been considered as being based on the (extended) principle of relativity on the one hand, and on the equivalence principle (equivalence of weight and inertia) on the other hand. Special Relativity, which is contained in General Relativity, has been considered as being based on the relativity principle on the one hand, and on the law of invariance of the (local) speed of light on the other hand. But the law of constant propagation of light can hardly be seen as being contained in the equivalence principle. It should therefore not come as a surprise to realize that the above assumptions regarding the empirical basis of Special and General Relativity are partly wrong. It is the (extended) relativity principle alone on which all of General and Special Relativity is built. From nothing else but that principle, Einstein's field equation of General and thus also of Special Relativity can be derived. This is done by operations in which the principles of conservation of mass and momentum (whose observation is required by the relativity principle for any observer at rest), and the covariant divergence of tensors play crucial roles. The simplified derivation of Einstein's field equation necessitates a reconsideration of Th. Kaluza's attempt of a unification of gravitation and electromagnetism (by the introduction of a fourth spatial dimension). Instead of introducing five new tensor elements $\mathbf{g^{40}}$, $\mathbf{g^{41}}$, $\mathbf{g^{42}}$, $\mathbf{g^{43}}$, $\mathbf{g^{44}}$ (as Kaluza did), five new tensor elements $\mathbf{T^{40}}$, $\mathbf{T^{41}}$, $\mathbf{T^{42}}$, $\mathbf{T^{43}}$, $\mathbf{T^{44}}$ are introduced that are an expression of conservation of charge. The result is stunning (even though electric force cannot be "transformed away"): Not only can Maxwell's equations be extracted, but the introduction of evenly distributed electric charge in the interior of a non-spinning spherical mass affects the $\mathbf{T^{\mu\,\nu}}$ tensor and hence also the metric tensor $\mathbf{g^{\mu\,\nu}}$ in the same way as if some mass were moving in the direction of a fourth spatial dimension. There are testable consequences, and, as a "by-product", there is a solution both to the Trouton-Noble and the Ehrenfest paradox.

Keywords: Einstein's field equation, relativity principle, equivalence principle, propagation of light, Kaluza's theory, fourth spatial dimension, brane, hyperspace, inner Schwarzschild solution, Reissner-Nordström solution, five-dimensional spacetime, Trouton-Noble paradox, Ehrenfest paradox, negative mass, cosmological constant, De-Sitter universe

I. Introduction

Einstein's field equation has been derived in various ways. Einstein himself first derived it from the equivalence principle and the extended relativity principle, which he used as starting points [see L. Susskind/A. Cabanes (2023), Lecture 9, pp. 295-329, especially p. 320, who followed Einstein's path]. There is also a derivation first found by D. Hilbert that uses the action or variation principle [see A. Einstein (1916), p. 167: "*The general theory of relativity has recently been given in a particularly clear form by H.A. Lorentz and D. Hilbert, who have deduced its equations from one single principle of variation*.") But it, too, needs further assumptions (see A. Einstein, op. cit., §§ 2 and 3, p. 169).

A derivation of Einstein's field equation solely from the relativity principle, without the use of the principle of variation and without the equivalence principle or the law of the invariance of the speed of light, is something new – and simple.

II. The relativity principle of General Relativity in mathematical terms

1) What the (extended) relativity principle is, and how it is related to the principles of conservation of mass and momentum

The extended relativity principle reads as follows: Any observer who finds himself in a position in which no force acts on him or her may consider himself or herself at rest, with all laws of physics, especially the law of conservation of mass and momentum, still valid.

In A. Einstein's (2018) words:

"In an example worth considering, the gravitational field has a relative existence only in a manner similar to the electric field generated by magneto-electric induction. Because for an observer in free-fall from the roof of a house there is during the fall—at least in his immediate vicinity—no gravitational field. Namely, if the observer lets go of any bodies, they remain, relative to him, in a state of rest or uniform motion, independent of their special chemical or physical nature. The observer, therefore, is justified in interpreting his state as being 'at rest.'"

As a consequence, this principle claims: If objects around the observer (who considers himself or herself as being at rest) are in accelerating motion (electromagnetic forces that could give rise to accelerations shall be absent), it is because of the curvature of spacetime only. This is why the accelerating motions of objects at some distance from the observer do not obstruct his claim of being at rest. The (extended) relativity principle is thus interwoven with the principle of conservation of mass and momentum. The way how exactly the (extended) relativity principle finds its way into Einstein's field equation of General Relativity will be revealed later on.

In order to give the principle of conservation of mass and momentum (which, as has just been announced, is to play an integral role in the relativity principle) a mathematical expression, four vectors must be formed in a four-dimensional diagram-space (**t,x,y,z**), and the covariant divergence (see below) of each of the vectors must be zero. The zero-divergence of one vector is an expression of the conservation of mass over time, the zero-divergence of a second vector is an expression of conservation of the **x**-component of momentum over time, the zero-divergence of a third vector is an expression of conservation of the **y**-component of momentum over time, and the zero-divergence of a fourth vector is an expression of the conservation of the **z**-component of momentum over time.

This translates into:
(1)

$$\nabla_\mu A^\mu = \delta_\mu A^\mu + \Gamma^\mu_{\alpha\mu} A^\alpha + \Gamma_{\alpha\mu} A^{\mu\alpha} = 0$$

and

(2)

$$\nabla_\mu B^\mu = \delta_\mu B^\mu + \Gamma^\mu_{\alpha\mu} B^\alpha + \Gamma_{\alpha\mu} B^{\mu\alpha} = 0$$

and
(3)

$$\nabla_\mu C^\mu = \delta_\mu C^\mu + \Gamma^\mu_{\alpha\mu} C^\alpha + \Gamma_{\alpha\mu} C^{\mu\alpha} = 0$$

and
(4)

$$\nabla_\mu D^\mu = \delta_\mu D^\mu + \Gamma^\mu_{\alpha\mu} D^\alpha + \Gamma_{\alpha\mu} D^{\mu\alpha} = 0$$

A, **B**, **C** and **D** are each vectors with four (contravariant) components. (In curved space or when axes of a coordinate system are not rectangular, there is no longer a unique way of determining a component of a vector. Instead, there are two ways. One way is called "contravariant", the other way is called "covariant".) Consequently, the general index **μ** runs from 0 to 3, and so does the general index **alpha**. The individual index 0 denotes the temporal coordinate **t**, the individual index 1 denotes the spatial coordinate **x**, the individual index 2 denotes the spatial coordinate **y**, the individual index 3 denotes the spatial coordinate **z**. The Einstein-summation-convention is applied to all variables, and thus also to **μ** and **alpha**. The **lambda**-sign is the Christoffel-symbol.

The equations say that the covariant divergence (and not necessarily the *ordinary* divergence **delta**$_\mu$) of each of the four vectors is zero. The covariant divergence is distinguished from the ordinary divergence, insofar as it throws out the effects of a "funny", for instance, curved coordinate system that might be used in ordinary, Euclidean space, or, conversely, the effects of "funny" space, that is, curved space, and of "funny", that is, location-dependent time, which might exist even when an ordinary, that is, non-curved coordinate system like a Cartesian system is used (see A. Trupp, 2022). The latter is because an arrangement of stationary meter sticks (laid end-to-end) and clocks in the vicinity of a gravitating mass might yield a mesh-system that *does* constitute a "funny" coordinate system – in comparison with a hypothetical situation of an arrangement of stationary meter-sticks and clocks extending over the same volume of space, but with *no* gravitating mass in the center. The logical possibility of such a phenomenon to exist had been revealed by B. Riemann (1854/1873).

[See d. Fleisch (2012), chapter 5.7, p. 149: "*So if you want to evaluate the changes in vector fields expressed in non-orthogonal coordinates, you have to account for possible changes in the basis vectors. Properly accounting for these changes means that the result of the differentiation process will retain the tensor characteristics of the original object. Fortunately, there's a way to account for any change in the basis vectors ... . That process, called the 'covariant derivative", is described in the next section of this chapter.*"]

In other words: Given an ordinary system of coordinates is chosen, our four equations are equivalent to saying that the principle of conservation of mass and momentum is conserved whenever those accelerations which are the direct or indirect result of the distortions (if any) of meter-sticks and of clock-ticking rates are removed from the picture.

2) Why the principles of conservation of mass and momentum are guaranteed by the zero-covariant divergence of the four vectors that make up the $T^{\mu\ nu}$-tensor

In order to get a deeper understanding of *why* the principle of conservation of mass and momentum is given an expression by the combination of those four equations, one must be aware of the fact that a vector forms a vector-*field* whenever it depends on coordinates. If it forms a field, it can be represented by field-lines, the density of which in diagram-space is an expression of the magnitude of the vector. The field-lines of each vector may extend over the whole diagram-space, or just over limited volumes of it. At places in diagram-space where the field lines of a vector stop or start, the divergence of the vector is different from zero. At places where they don't, the divergence is zero. In the former case, we are facing a situation in which the principle of conservation of mass and momentum is violated, provided the four-dimensional vector in four-dimensional, Cartesian **t**,**x**,**y**,**z**-diagram-space is an expression of mass or momentum.

For an illustration, one should, for instance, imagine that a solid sphere at rest in Euclidian space (that is, in real three-dimensional space) would abruptly disappear into nothingness. A representation of that event – occurring in three-dimensional Euclidean space – shall be provided in a Cartesian **t**-,**x**-,**y**-,**z**- diagram. Mass density of objects shall be a function of **t,x,y** and **z**. As the mass elements of the solid sphere do not, by arrangement, move in space, the **x**-,**y**- and **z**- values of any mass points of the solid sphere are constants, whereas the **t**-values are not. By using time **t** as a fourth coordinate in diagram-space, mass-density is no longer a scalar, but has turned into a 4-vector. When the mass is sitting still, the direction of that vector is strictly parallel to the **t**-axis of the diagram. We then have vertical field lines inside the sphere (which is a cylinder in the four-dimensional diagram-space), provided that **t** is the vertical axis in the diagram. The density of the field-lines in diagram-space is an expression of the magnitude of mass density. This bundle of vertical field lines in Cartesian **t**-,**x**-,**y**-,**z**-diagram-space would have an abrupt end. The (ordinary) divergence of that vector-field would thus be different from zero (at this point in the four-dimensional **t**-,**x**-,**y**-,**z**-diagram).

Hence, a non-zero divergence of any of the vectors **A**, **B**, **C** or **D** would be a telltale sign of a violation of the principle of conservation of mass and momentum.

Next, we ask whether or not it is permissible to consolidate these four equations into a single equation in which the covariant divergence of a symmetrical 4 x 4 tensor $\mathbf{T}^{\mu\ nu}$ is set to zero. As a result of the (covariant) divergence the we have formed, we would then have a 4-vector (which we call **A**) all of whose four (contravariant) components **nu** are zero:
(5)

$$\nabla_\mu T^{\mu\nu} = \delta_\mu T^{\mu\nu} + \Gamma^\mu_{\alpha\mu} T^{\alpha\nu} + \Gamma^\nu_{\alpha\mu} T^{\mu\alpha}$$

$$= \delta_\mu T^{\mu\nu} + \frac{g^{n\mu}}{2}\left(\frac{\delta g_{\alpha n}}{\delta x^\mu} + \frac{\delta g_{\mu n}}{\delta x^\alpha} - \frac{\delta g_{\alpha\mu}}{\delta x^n}\right) T^{\alpha\nu} + \frac{g^{n\nu}}{2}\left(\frac{\delta g_{\alpha n}}{\delta x^\mu} + \frac{\delta g_{\mu n}}{\delta x^\alpha} - \frac{\delta g_{\alpha\mu}}{\delta x^n}\right) T^{\mu\alpha} = A^\nu = 0$$

This would be an expression of the principle of conservation of mass and momentum. All general indices run from 0 to 3 and refer to the system of coordinates used. If polar coordinates are used, we have 0=**t**, 1=**r**, 2= **theta**, 3= **phi**. The term **x** is a generalized expression of these coordinates. Hence, in case of polar coordinates, we have: $\mathbf{x^0=t}$, $\mathbf{x^1 = r}$, $\mathbf{x^2= theta}$, $\mathbf{x^3= phi}$.

The answer to the question raised is in the positive. Any symmetrical 4 x 4 -tensor can be thought of as being made up of four four-dimensional vectors (*non*-symmetrical 4 x 4 tensors cannot be said to be represented by four vectors, since horizontal lines then yield different packages of four elements compared to the packages yielded by vertical columns). Hence, the only thing we have done in (5) is to give the four vectors **A**, **B**, **C**, **D** new names, these names being $\mathbf{A=T^{\mu 0}}$, $\mathbf{B=T^{\mu 1}}$, $\mathbf{C=T^{\mu 2}}$, $\mathbf{D= T^{\mu 3}}$. Of course, we now get a (zero-) *vector* with four zero-components as a result of the divergence, and no longer four separate, unrelated numbers (each of them being zero). This does no harm.

III. Determination of the 16 elements of the symmetrical 4 x 4 tensor $\mathbf{T^{\mu\ nu}}$

Let us now determine the meanings of the 16 elements of the tensor $\mathbf{T^{\mu\ nu}}$.

We go back to the example of the solid sphere at rest in Euclidean space. If nothing else than the stationary solid sphere is present, the symmetrical 4 x 4 tensor $\mathbf{T^{\mu\ nu}}$ has to display a horizontal row and also a vertical column, in which, in both cases, one element is mass-density, and the other three elements must be zero (as are all the other elements). Morever, when forming the divergence of that vector **rho**, 0, 0, 0, mass density (= **rho**) must be the first element both in the horizontal row line and in the vertical column. Only then is it that this element is differentiated by **dt** and not by **dx**, **dy** or **dz**, regardless of whether we use the horizontal line or the vertical column (we are using Cartesian coordinates for the moment). If **rho** were not differentiated by **dt**, but by **dx**, **dy** or **dz**, we could get a non-zero result for the divergence, even if the law of conservation of mass and momentum is conserved. This allows only one single place (in the $\mathbf{T^{\mu\ nu}}$-tensor) for the mass density: $\mathbf{T^{00}}$.

On the basis of this knowledge, the elements $\mathbf{T^{01}}$, $\mathbf{T^{02}}$, $\mathbf{T^{03}}$ must represent mass-flux [in units of "kg/(sec m²) "] in the **x**-, the **y**-, and in the **z**-direction, respectively. Only then is it that
(6)

$$\frac{\delta T^{00}}{\delta t} + \frac{\delta T^{01}}{\delta x} + \frac{\delta T^{02}}{\delta y} + \frac{\delta T^{03}}{\delta z} = 0$$

can hold true [with each summand having the dimension of "kg/(sec m³)"].

Moreover, $\mathbf{T^{10}}$, $\mathbf{T^{20}}$, $\mathbf{T^{30}}$ must represent the same "mass-flux" in the **x**-, the **y**-, and the **z**-direction [in units of kg/(sec m²)]. Only then is it that
(7)

$$\frac{\delta T^{00}}{\delta t} + \frac{\delta T^{10}}{\delta x} + \frac{\delta T^{20}}{\delta y} + \frac{\delta T^{30}}{\delta z} = 0$$

can hold true [with each summand having the dimension of “kg/(sec m^3)”].

Similarly, $\mathbf{T}^{11}$, $\mathbf{T}^{21}$, $\mathbf{T}^{31}$ must represent *momentum*-flux through a surface whose normal points in the **x**-direction [in units of kg m/(sec m^2 sec) = kg/(sec^2 m)], with $\mathbf{T}^{11}$ giving the **x**-component of that momentum-flux, $\mathbf{T}^{21}$ giving the **y**-component, and $\mathbf{T}^{31}$ giving the **z**-component of that flux. Only then is it that
(8)

$$\frac{\delta T^{01}}{\delta t} + \frac{\delta T^{11}}{\delta x} + \frac{\delta T^{21}}{\delta y} + \frac{\delta T^{31}}{\delta z} = 0$$

can hold true [with each summand having the dimension of “kg/(sec^2 m^2)”].

Similarly, $\mathbf{T}^{12}$, $\mathbf{T}^{22}$, $\mathbf{T}^{32}$ must represent momentum-flux through a surface whose normal points in the **y**-direction [in units of kg m/(sec m^2 sec) = kg/(sec^2 m)], with $\mathbf{T}^{12}$ giving the **x**-component of that momentum-flux, $\mathbf{T}^{22}$ giving the **y**-component, and $\mathbf{T}^{32}$ giving the **z**-component of that flux. Only then is it that
(9)

$$\frac{\delta T^{02}}{\delta t} + \frac{\delta T^{12}}{\delta x} + \frac{\delta T^{22}}{\delta y} + \frac{\delta T^{32}}{\delta z} = 0$$

can hold true [with each summand having the dimension of “kg/(sec^2 m^2)”].

Finally, $\mathbf{T}^{13}$, $\mathbf{T}^{23}$, $\mathbf{T}^{33}$ must represent momentum-flux through a surface whose normal points in the **z**-direction [in units of kg m/(sec m^2 sec) = kg/(sec^2 m)], with $\mathbf{T}^{13}$ giving the **x**-component of that momentum-flux, $\mathbf{T}^{23}$ giving the **y**-component, and $\mathbf{T}^{33}$ giving the **z**-component of that flux. Only then is it that
(10)

$$\frac{\delta T^{03}}{\delta t} + \frac{\delta T^{13}}{\delta x} + \frac{\delta T^{23}}{\delta y} + \frac{\delta T^{33}}{\delta z} = 0$$

can hold true [with each summand having the dimension of “kg/(sec^2 m^2)”].

The meanings of the 16 elements of $\mathbf{T}^{\mu\ \mathrm{nu}}$ cannot change in case curvature of spacetime results in an acceleration of test objects (so that the covariant divergence is no longer the same as the ordinary divergence).

[As a consequence of their meanings, the three diagonal elements $\mathbf{T}^{11}$, $\mathbf{T}^{22}$, $\mathbf{T}^{33}$ are often described as “pressure”. This is because a wall (whose normal points in the **x**-, **y**-, or **z**-direction), constantly bombarded by particles that stop at the wall, would be subject to a pressure of the same magnitude, that is, the normal component of the momentum flux. With the meanings given to the 16 elements, Equation (5) is, strictly speaking, not an expression of energy conservation, but of mass conservation (since $\mathbf{T^{00}}$ is defined as mass density and not as energy density). However, it is a well known consequence of Special Relativity that mass is proportional to energy, with the factor of proportionality **k** in **E = kM** being $\mathbf{c^2}$.]

IV. The final step: From the tensor $\mathbf{T^{\mu\ nu}}$ to Einstein’s field equation

1) Einstein’s field equation without the cosmological constant

Next, we consider two symmetrical 4 x 4-tensors, $\mathbf{T^{\mu\ nu}}$ and $\mathbf{G^{\mu\ nu}}$, which we presume to share the same quality of a zero covariant divergence. We cannot yet say whether or not these two tensors, each of which can be represented in a Cartesian diagram by four vectors, are identical or at least proportional to each other. But things change as soon as we arrange that all elements of the two tensors are zero outside of any mass **M**. Then, given the fact that the four vectors representing the tensor $\mathbf{T^{\mu\ nu}}$ may assume any values and directions whatsoever in the interior of the masses, we realize that the two tensors MUST be proportional to each other, that is, $\mathbf{G^{\mu\ nu} = k\ T^{\mu\ nu}}$, with **k** being a constant. To make this evident, we imagine that all the lumps of matter distributed in space and time are made up of tiny grains separated from one another by differentially small distances only. Each of these grains forms a hair-thin tube in a four-dimensional, Cartesian **t**,**x**,**y**,**z**-diagram. Since these tubes can be thought of as being as thin as one likes them to be, any vector-field inside a hair-thin tube cannot but align itself with the tube, given that it cannot leave the tube, and given that its field-lines are not disrupted anyhere. [A similar, though not identical reflection with respect to the tensor $\mathbf{G^{\mu\ nu}}$ can be found in L. Susskind/A. Cabannes (2023), p. 320: “*A theorem can be proved that says there is no other tensor (up to a multiplicative factor) made up out of two derivatives acting on the metric that is covariantly conserved.*”]

Hence, presuming that the covariant divergence of $\mathbf{G^{\mu\ nu}}$ is zero, the tensor $\mathbf{G^{\mu\ nu}}$ must indeed be proportional to $\mathbf{T^{\mu\ nu}}$. In mathematical terms:
(11)

$$(\nabla_\mu G^{\mu\nu} = 0 \ \wedge \ \nabla_\mu T^{\mu\nu} = 0) \quad \Rightarrow \quad G^{\mu\nu} = kT^{\mu\nu}$$

Next, let us assume we would know that the covariant divergence of a new tensor
(12)

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R$$

which, too, shall be zero outside matter, is always zero. It, too, is proportional to $\mathbf{T^{\mu\ nu}}$. We could then convert (11) into:
(13)

$$(\nabla_\mu G^{\mu\nu} = 0 \;\wedge\; \nabla_\mu T^{\mu\nu} = 0) \;\Rightarrow\; G^{\mu\nu} = k_1(R^{\mu\nu} - \frac{1}{2}\, g^{\mu\nu}R) = k_2 T^{\mu\nu}$$

This is Einstein's field equation.

What remains to be done is, of course, the following: We have to show that the presumption of a zeroness of the new tensor outside matter does not lead to contradictions, and we have to prove that its covariant divergence is always zero.

Let's solve the first task. There are two equations which we know to be to be valid in tensor calculus. These equations are:

$$R = g_{\mu\nu} R^{\mu\nu} \qquad , \qquad g_{\mu\nu} g^{\mu\nu} = 4$$

We therefore get (**k_3** is another constant):
(14)

$$R^{\mu\nu} - \frac{1}{2}\, g^{\mu\nu}R = R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} g_{\mu\nu}\, R^{\mu\nu} = R^{\mu\nu} - 2R^{\mu\nu} = -R^{\mu\nu} = k_3 T^{\mu\nu}$$

Given that there is a proportionality between **$R^{\mu\ nu}$** and **$T^{\mu\ nu}$**, there is hence no contradiction in saying that every element of the tensor **$R^{\mu\ nu}$** is zero outside matter (and that, as a mathematical consequence, the scalar **R** is also zero outside matter). The absence of any contradictions persists even when we give **$R^{\mu\ nu}$** the meaning of the Ricci-tensor. [This shall not be proved here, but it shall be noted that our choice of the meaning of **$R^{\mu\ nu}$** is indispensable for the validity of a theorem presented in (15), which we will use immediately. One should note that a contradiction *would* exist if it were the metric tensor **$g^{\mu\ nu}$** that is said to be zero outside matter.]

Finally, we must solve the second task. We have to show that the covariant divergence of the new tensor is indeed zero in any chosen case. In order to accomplish this goal, we use the following theorem (that shall not be derived here) from tensor calculus:
(15)

$$\nabla_\mu R^{\mu\nu} = \frac{1}{2} g^{\mu\nu}\, \delta_\mu R$$

We have thus been able to replace the covariant divergence of the **$R^{\mu\ nu}$** -tensor by an expression which contains the ordinary divergence of the contracted Ricci-tensor (that is, the gradient of the Ricci-scalar).

Moreover, since the covariant divergence of any metric tensor **$g^{\mu\ nu}$** is zero, and since the covariant "divergence" of the contracted Ricci-tensor **R** is equal to its ordinary "divergence", that is, to the gradient of the Ricci-scalar, we can formulate (using the chain rule of differentiation):
(16)

$$\nabla_\mu(\frac{1}{2}g^{\mu\nu}R) = \frac{1}{2}(\nabla_\mu g^{\mu\nu})R + \frac{1}{2}g^{\mu\nu}\,\nabla_\mu R = \frac{1}{2}g^{\mu\nu}\,\nabla_\mu R = \frac{1}{2}g^{\mu\nu}\,\delta_\mu R$$

With the left sides of (15) and (16) thus being equal to each other, we get:
(17)

$$\nabla_\mu R^{\mu\nu} - \nabla_\mu(\frac{1}{2}g^{\mu\nu}R) = \nabla_\mu\,(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R) = 0$$

This had to be proved (it is also common knowledge).

We have thus cleared both of the two caveats from Einstein's field equation. It now reads:
(18)

$$R^{\mu\nu} - \frac{1}{2}\,g^{\mu\nu}R = k\,T^{\mu\nu} = -\frac{8\pi G}{c^2}\,T^{\mu\nu}$$

The constant **k** is determined by setting it equal to $\mathbf{R^{00}/T^{00}}$ or $\mathbf{-R^{00}/T^{00}}$ (see above, where we obtained $\mathbf{-R^{\mu\,nu}=kT^{\mu\,nu}}$). Due to its chosen meaning as an element of the Ricci-tensor, the numerical value of $\mathbf{R^{00}}$ can be given a negative or a positive sign, and so can the constant **k**. We decide to give $\mathbf{R^{00}}$ a positive sign, so **k** has to have a negative sign. The sign of $\mathbf{T^{00}}$ is fixed as positive (with an exception that will be addressed below). **G** is Newton's gravitational constant (and not the contracted $\mathbf{G^{\mu\,nu}}$ -tensor).

One should note that, because of (14), Einstein's field equation can also be written as:
(19)

$$R^{\mu\nu} - \frac{1}{2}\,g^{\mu\nu}R = k\,T^{\mu\nu} \;\Leftrightarrow\; -\frac{1}{2}\,g^{\mu\nu}R = 2k\,T^{\mu\nu} \;\Leftrightarrow\; g^{\mu\nu}R = \frac{32\pi G}{c^2}\,T^{\mu\nu}$$

2) Einstein's field equation with the inclusion of the cosmological constant

As is commonly known, Einstein added a negative summand **lambda** $\mathbf{g^{\mu\,nu}}$ on the left-hand side of his field equation:
(20)

$$R^{\mu\nu} - \frac{1}{2}\,g^{\mu\nu}R - \lambda g^{\mu\nu} = kT^{\mu\nu}$$

Since the covariant divergence of **lambda** $\mathbf{g^{\mu\,nu}}$ is zero if **lambda** is a constant, it follows from what we said above that the proportionality of the tensor $\mathbf{T^{\mu\,nu}}$ and the tensor that forms the left-hand side of Einstein's field equation is not affected by the introduction of an additional summand on the left-hand side. But given the restrictions we set up (according to which **R** and $\mathbf{R^{\mu\,nu}}$ have to be both zero outside of matter), and given that $\mathbf{g^{\mu\,nu}}$ is nowhere zero, this requires that all of space is now filled with matter (of minuscule density at least), if

lambda is non-zero. Otherwise we would, outside matter, have a zero right-hand side and a non-zero left-hand-side of Einstein's equation. In other words: **lambda $g^{\mu\, nu}$** is an expression of mass density. We thus write (the negative summand **lambda $g^{\mu\, nu}$** is transferred to the right-hand side):
(21)

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu}R = kT^{\mu\nu}_{total} = kT^{\mu\nu}_{ord} + \lambda g^{\mu\nu} = kT^{\mu\nu}_{ord} + kT^{\mu\nu}_{cosm}$$

The subscript "ord" stands for ordinary matter, the subscript "cosm" for cosmological matter. We realize that whenever $\mathbf{T}^{\mu\, nu}$ is used without subscript [as in (20)], it means $\mathbf{T}_{ord\mu}{}^{nu}$.

In case no other mass than that expressed by **lambda $g^{\mu\, nu}$** exists, (21) turns into:
(22)

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu}R = \lambda g^{\mu\nu} = kT^{\mu\nu}_{cosm}$$

[See L. Susskind/A. Cabanes (2023), p. 327: "*We haven't said anything about the cosmological constant – whether it exists or not – because it can be thought of as part of* $\boldsymbol{T}^{\mu\, nu}$ *. From this point of view, the cosmological constant is an extra tensor term on the right hand side of equation (...). We could denote it* $\boldsymbol{T}_{cosmological}{}^{\mu\, nu}$ *. And that would not change the look of Einstein's equation. If it was indeed a scalar, we could write it* $\boldsymbol{T}_{cosmological\mu}{}^{nu}$ *=* ***lambda*** $\boldsymbol{g}^{\mu\, nu}$*.*"]

Let us now consider a situation in a region of flat spacetime where $\mathbf{g^{00}=1=g_{00}}$. By arrangement, the tensor element $\mathbf{T}_{cosm0}{}^{0}$ is constant over space and over time. Also by arrangement, it is now endowed with the capacity of being either numerically positive or negative. Then (22) turns into (**rho** is mass density):
(23)

$$(R^{00} - \frac{1}{2} g^{00}R = -R^{00} = \lambda g^{00} = -\frac{8\pi G}{c^2} T^{00}_{cosm} = -\frac{8\pi G}{c^2} \rho_0 \wedge g^{00} = 1$$

$$\wedge \lambda \neq 0 \wedge \frac{dr}{dt} > 0) \Rightarrow (\lambda = -\frac{8\pi G}{c^2}\rho_0 \wedge \rho_0 < 0 \wedge T^{00}_{cosm} < 0 \wedge R^{00} < 0)$$

That is to say: If a non-zero **lambda** is to lead to a solution of Einstein's field equation that describes an expanding universe (as is the case with cosmic variant of the Schwarzschild solution), it is inevitable to give mass density **rho** a negative sign. With our choice of a negative **k** in Einstein's field equation, **lambda** is numerically positive, whereas $\mathbf{T}_{cosmological}{}^{00}$ and **rho** are numerically negative independently of our choice for **k**. An *expansion* of space is not brought about by an endless cloud (in which point masses do not interact other than by gravity) of *positive* matter density, for instance, by a cloud of electromagnetic fields. Instead, such a cloud bring about a *contraction* of space (**dr/dt<0**) as a consequence of Einstein's field equation.

The “negativeness” of **rho** is obscured in almost all descriptions that identify Einstein’s cosmological constant with “dark energy” or “vacuum energy”. Moreover, quite often $\mathbf{g^{00}}$ is postulated as -1, and not as 1. Then, with a positive **rho**, the constant **lambda** would be positive in (11e), and $\mathbf{R^{00}}$, too, would be positive. A positive $\mathbf{R^{00}}$ would, in turn, lead to a positive $\mathbf{T^{00}}$ in Einstein’s field equation. This, however, would be incompatible with $\mathbf{d^2r/dt^2 > 0}$. That is to say: Even if $\mathbf{g^{00}}$ were equal to -1, **rho** would have to be negative. Then **lambda** would be negative, and so would $\mathbf{R^{00}}$ and $\mathbf{T_{cosm0}{}^{0}}$. Only this outcome – and not the outcome which one gets when setting **rho>0** – is compatible with **dr/dt>0**.

A further scrutiny confirms this result. The cosmic variant of the Schwarzschild solution, based on a non-zero **lambda** (De Sitter space), reads:
(24)

$$d\tau^2 = \left(1 - \frac{H^2r^2}{c^2}\right) dt^2 - \frac{1}{c^2\left(1 - \frac{H^2r^2}{c^2}\right)} dr^2 - \frac{r^2}{c^2}(d\theta^2 + \sin^2\theta \; d\varphi^2)$$

H is Hubble’s constant; **r** is radial distance measured in circumference of a circle around the Milky Way, divided by 2 **pi**.

When comparing (24) with the inner Schwarzschild solution for a spherical mass [see (38) below], one can show that both solutions merge into one and the same thing in a special situation of the following kind: An observer shall be at the center of a spherical body. As will be mentioned in greater detail below, the inner Schwarzschild solution says a meter stick at rest at the center of the spherical mass does not undergo a length-contraction for an outside observer. In the inner Schwarzschild solution, one can therefore exchange the roles of the stationary outside observer and that of the stationary observer at the center. If, both in the inner Schwarzschild solution and in the cosmic variant, $\mathbf{dtau^2}$ is exchanged by $\mathbf{ds^2}$, and if all differentials except **ds** and **dr** are set to zero, we get from the two solutions (the inner Schwarzschild solution for a spherical mass on the one hand, and the cosmic variant describing De-Sitter space on the other hand):
(25)

$$ds^2 = \frac{1}{1 - \frac{H^2r^2}{c^2}} dr^2 = \frac{1}{1 - \frac{2r^2GM}{c^2R_0^3}} dr^2 = ds^2 \quad \Rightarrow \quad \frac{dr}{dt} < 0$$

$\mathbf{2GM/R_0^3}$, appearing in the inner Schwarzschild solution and thus on the right-hand side of (25), is an expression of positive (!) mass density. Let us imagine $\mathbf{R_0}$ exceeds all limits. Then we are permitted to apply the cosmic variant (left-hand side of the equation) to this situation, and we may set the equal sign between the two sides. As a consequence, **H**, too, must be an expression of mass density. Because of the numerical positiveness of $\mathbf{2GM/R_0^3}$ and thus of mass density, our equation (25) describes a situation in which space is not expanding, but contracting: Test objects fall towards the center. The cosmic variant of an equation yielding the velocity of this free fall reads [the primed reference frame in (26) is that of a very distant, stationary observer who is imagined to be tethered to the Milky Way. His or her time is **tau** or

t'. The term **dR** is difference in radial distance with respect to radially oriented, stationary meter sticks which the tethered observer holds in his or her hands]:
(26)

$$v_{escape} = \frac{dr}{dt} = Hr(1 - \frac{H^2 r^2}{c^2}) \quad \wedge \quad v'_{escape} = \frac{dR}{d\tau} = \frac{dR}{dt'} = Hr$$

If we use that equation (which was derived from the cosmic variant of the Schwarzschild solution) and not the inner Schwarzschild solution for a determination of the velocity of free fall, we have to choose between a positive or a negative sign for **H**. In order for **dr/dt** to be negative, that is, in case of a positive mass density, that sign has to be negative, too. Conversely, the sign of **H** has to be positive and mass density has to be negative, if **dr/dt** is to be positive. That is the case when it comes to cosmic expansion.

Negative mass or energy, longed for by a technologically advanced future civilization in order to keep "wormholes" open, is thus not as exotic as believed. It's all over the place. With **lambda** (in SI-units) being 1.1056 x 10^{-52} (as a result of measurements of the Hubble constant), **rho** is [according to (11e)]: -5.9 x 10^{-27} kg/m³. In order to contain 1 kg of evenly distributed negative mass, a space-volume of cube-shape must have a side-length of 0.55 x 10^9 m = 550000 km.

[The use of the wrong sign for **rho** has had drastic consequences for cosmology: In order to find out whether or not the universe is eternally expanding, Newtonian physics has been applied (to start with). All the visible matter of the universe is imagined to be exploding. The role of kinetic energy in Newtonian physics has been taken over by cosmic expansion. The mutual gravitational attraction of the exploding parts of the visible universe tries to decelerate the expansion. In case all parts have a velocity that is higher than their escape velocity, the expansion will never stop. In case all parts have a velocity lower than their escape velocity, expansion will eventually come to a halt, and will then be reversed.

Cosmologists have so far believed that vacuum energy, due to its apparently positive sign, is helping to pull back galaxies by means of its gravitational action on these objects (despite its being responsible for the outward "pressure"). But that's clearly wrong (according to Einstein's field equation): The repulsive action of negative masses is the only action that can be expected from them. Moreover, (24), (25) and (26) show: Evenly distributed *negative* mass brings about accelerations of space (or cosmic expansion) that can be considered as a time-reversal of those accelerations of space which are brought about by evenly distributed, positive mass. (As regards the concept of "accelerating motion of space", see also below.)

We therefore find: By wrongly assuming that vacuum mass or energy is positive, cosmologists must inevitably come to the wrong conclusion that the "force" of expansion and the "force" of contraction exactly cancel each other.

On top, there is *ordinary* positive mass, which is also evenly distributed in space. This additional mass cannot change the wrong picture: In the Newtonian model, the ordinary positive mass receives the same outward kick per kg as the vacuum mass does. The apparent relationship of sameness in magnitude between outward kick and inward "force" is therefore the same as it is for the vacuum mass.

What astronomers, to their surprise, call a mysterious equality of inward and outward "forces", or a universe that mysteriously finds itself very close to its "critical mass density", is a mere artefact. In reality, the universe is expanding without being obstructed by anything, as long as escaping galaxies obey Hubble's law (which shows that binding forces are negligible.]

Consequently, no "flat" universe can be expected. The cosmic variant of the Schwarzschild solution rather tells us that, for a universe in which vacuum energy or vacuum mass prevails over any other sort of mass or energy, the universe is a far shot from being flat. Instead, it can best be described by the cosmic version of Flamm's parabola: In a three-dimensional model, it forms a paraboloid of revolution like the surface of rotating water in a glass does. We and the Milky Way are at the lowest point of the surface. The number of radially oriented, stationary meter sticks laid end-to-end, starting at our position and reaching out to a circle whose radius is measured in length of circumference divided by 2 **pi**, exceeds what we would expect in flat space. This is what the left-hand side of (25) tells. The quotient of **ds/dr** gives the rate of length-contraction of stationary, radially oriented meter-sticks as a function of **r**. That phenomenon becomes the more pronounced the closer the end of the line of meter-sticks gets to the Milky Way's cosmic event horizon. Given the phenomenon of escaping galaxies is an empirical fact, we have all reason to believe that this is also true for our real universe, in which ordinary matter exists beside negative vacuum matter.

To recapitulate: Our derivation of Einstein's field equation does not give rise to a mere "*interpretation*" according to which the additional summand **lambda** $\mathbf{g}^{00}$ in Einstein's field equation is an expression of mass- or energy-density. Instead, it makes this recognition *compelling*. Moreover, the numerical value of that mass- or energy-density has a negative sign.

V. The demotion of the principle of the invariance of the speed of light

We are now about to show the following: The law of the invariance of the speed of light is not, as is wrongly assumed in articles or books on Special Relativity, a foundation, that is, a second starting point, of Special Relativity. [See A. Einstein (1905/1952), p. 37/38: "*... the same laws of electrodynamics and optics will be valid for all frames of reference for which the equations of mechanics hold good. We will raise this conjecture (the purport of which will hereafter be called the 'Principle of Relativity') to the status of a postulate, and also introduce another postulate, which is only apparently irreconcilable with the former, namely, that light is always propagated in empty space with a definite velocity c which is independent of the state of motion of the emitting body. These two postulates suffice ...*"; see also A. Einstein (1961), Chapter VII, p. 19/20: *"In view of this dilemma there appears to be nothing else for it than to abandon either the principle of relativity or the simple law of the propagation of light in vacuo. ... As a result of an analysis ... it became evident that in reality there is not the least incompatibility between the principle of relativity and the law of* *propagation of light, and that by systematically holding fast to both these laws a logically rigid theory could be arrived at. This theory has been called the special theory if relativity* ..."; see also R. Sexl / H.K. Schmidt (1979), Chapter 7.4, p. 73.] Instead, the law of the invariance of the speed of light is DERIVED from Einstein's field equation and hence from the relativity principle. The relativity principle thus is the ONE AND ONLY foundation of Special (and General) Relativity.

In order to prove this, we turn our attention to the (outer) Schwarzschild solution of Einstein's field equation, valid for a non-spinning, spherical mass. It comes in the form of a "line-element" and reads [in polar spatial coordinates, $\mathbf{r_s}$ is the Schwarzschild radius (at which the local escape velocity is **c**), **tau** is the proper time of an observer in the gravity field, **t** is the time of a stationary observer far away from the spherical mass, **r** is circumference of a circle around the center of the spherical mass, divided by 2 **pi,** and is the same for both observers]:
(27)

$$d\tau^2 = \left(1 - \frac{r_s}{r}\right) dt^2 - \frac{1}{c^2\left(1 - \frac{r_s}{r}\right)} dr^2 - \frac{r^2}{c^2}(d\theta^2 + \sin^2\theta \; d\varphi^2)$$

If we confine ourselves to motions of observers and test-objects in the equatorial plane and in a radial or anti-radial direction (so that **d phi** and **d theta** are both zero), the equation turns into:
(28)

$$d\tau^2 = \left(1 - \frac{r_s}{r}\right) dt^2 - \frac{1}{c^2\left(1 - \frac{r_s}{r}\right)} dr^2$$

For an interval of proper time **tau** of an observer who finds himself or herself far away from the spherical mass (as is the case for the other observer whose time is **t**), we thus get:
(29)

$$d\tau^2 = dt^2 - \frac{1}{c^2} dr^2$$

This is Minkowski's line element of spacetime in Special Relativity: Even though a temporal interval between two point-events occurring at the same spatial place for one observer can be measured differently in length by another observer, the difference between the squared temporal interval and the squared spatial interval is always the same for all observers.

For a photon or a light-pulse traveling at speed **c** in the coordinate-system of an observer (whose time is **t**) at rest far away from the spherical mass, the proper time **tau** of the photon that elapses between two point-events occurring at the same place in the photon's frame of reference is [according to (29)]:

(30)

$$d\tau^2 = 0$$

This zero-length of the proper time interval **d tau** is absolute in a sense that in any reference frame in which an observer uses **t** as his time, **d tau** – that is, the proper time interval of the photon – is zero. As regards the velocity of the photon (far away from the spherical mass) in any unprimed reference frame of an observer who considers himself as being at rest, we hence get from (29):
(31)

$$v_{photon}^2 = \frac{dr^2}{dt^2} = c^2$$

This constitutes the law of the invariance of the speed of light in Special Relativity.

[In a gravitational field around a non-spinning spherical mass, the Schwarzschild solution yields a lower speed of light (that even depends on whether the trajectory is radial or tangential), but only for the outside observer who sits far away; for a local observer, the speed is still **c**. See L. Flamm (1916/2015) : "*Expressed in coordinates, which are mere parameters in the formulation of the gravitational field, the speed of light is by no means constant; in fact, it has different values in different directions even at the same location. But, when measured with material rods and clocks, the propagation of light also appears homogeneous and isotropic in a gravitational field*."]

Hence, the empirical corroboration of the law of the invariance of the speed of light is an empirical corroboration of General Relativity and not a starting-point, neither of General nor or Special Relativity.

VI. The demotion of the equivalence principle

1) How general should General Relativity be?

It is commonly thought that the equivalence principle is, besides the relativity principle, the second of two cornerstones of General Relativity [see A. Einstein (1921), p. 247: "*The general theory of relativity owes its existence in the first place to the empirical fact of the numerical equality of the inertial and gravitational mass of bodies, for which fundamental fact classical mechanics offered no interpretation. ... As a result of this, the general theory of relativity, which is based on the equality of inertia and weight, provides a theory of the gravitational field*."]

A. Einstein (1916a/1952), p. 114, expanded on this "cornerstone" as follows, when he compared a reference frame **K** at rest in a gravity field with another reference frame **K'** subject to acceleration by an ordinary force:

"Therefore, from the physical standpoint, the assumption readily suggests itself that the systems K and K' may both with equal right be looked upon as 'stationary', that is to say, they have an equal title as systems of reference for the physical description of phenomena."

But in order for the assertion "I have a title as a system of reference" not to be tautological, that is, a consequence of a mere definition of "system of reference" and of one's liberty to define all strange motions of objects that apparently violate the principles of conservation of mass and momentum simply as results of curvatures of spacetime, that system must possess the property of not being privileged over other systems that are based on the same curvature of spacetime (see also just below).

To put it differently: Since it is the covariant and not the ordinary divergence of the vectors which are expressions of conservation of mass and momentum that is set to zero, one could search and try any curvature of one's desire until one has found one that fits and "mends" the apparent violation of the principle. For the curvature of spacetime gets its shape by the criterium that it has to mend an apparent violation of the principles of conservation of mass and momentum. This resembles the right-hand side of a commerical balance-sheet, whose sum is surely equal to that on the left-hand side, simply because the last entry ("equity") on the right-hand side of the balance-sheet is added in order to bring this equality about. In other words: The balance sheet is always prepared in such a way that the sum of the assets equals the sum of the liabilities plus equity. Similarly, the curvature of spacetime "found" by means of a solution of Einstein's field equation is always the curvature needed to make the apparent violation of the principles of conservation of mass and momentum disappear.

Moreover, due to the mathematical nature of vectors and their zero-covariant divergence, there are, as a mathematical necessity, other frames of reference in which the covariant divergence of the four vectors that make up Einstein's field equation is zero, too (for the same constellation of objects and their motions in spacetime). Whether or not observers in these reference frame would have a reciprocal experience with respect to relativistic effects would *not* be guaranteed, but would be of no interest anyway. So everything that is to be known seems to be certain apriori, and not aposteriori.

To give an example: Imagine an observer on a merry-go-round who considers himself or herself at rest. It would be no problem to "transform away" all violations of the principle of conservation of mass und momentum. One would simply have to postulate a complicated "curvature of spacetime" that involves the whole universe. A success in transforming away all violations of the principles of conservation of mass and momentum would be guaranteed without any chance of failing. And there would be, as a mathematical necessity, other frames of reference, in all of which the principles of conservation of mass and momentum would be observed with regard to the same masses and their motions. However, that would not be what we want.

A chance of failing (which is necessary for any hypothesis that is to be scientific) is only brought about as soon as one asserts that any other frame of reference (in which the conservation of mass and momentum is guaranteed) has to be reciprocal with respect to relativistic effects. For reasons of symmetry (see just below), a reciprocity of relativistic effects can (only) be expected to exist among those frames of reference (in all of which the principle of conservation of mass and momentum is observed for the same masses and their motions) in which an observer does not feel a force on him or her. That's the contents of the extended relativity principle. It postulates, in a nutshell, that there is no such thing as absolute rest. But in case relativistic effects were not reciprocal among two reference frames described, one of the two frames would be a privileged one. Absolute rest would thus exist, contrary to what the relativity principle postulates. Absolute rest would constitute a breaking of a kind of symmetry, as no reason could be given why one reference frame should be picked by nature as the one out of the many (in all of which an observer feels no force on him or her). Conversely, given the infinitely large number of possible rest-frames (for a given constellation of masses and their motions), a reciprocity of the described kind would be highly coincidental and would not be backed by symmetry arguments if literally all observers could consider themselves as being at rest.

More precisely: In case literally all observers were entitled to consider themselves at rest at the origin of a coordinate system, there would be more than one solution of Einstein's field equation, namely infinitely many, for one and the same constellation of masses and their motions. As will be shown below, the magnitude of relativistic effects (length contraction and time dilation) is a function of the speed of local flows of space-cells past clocks and meter sticks, regardless of whether these flows of space are real or only imagined. In reference frames in which an observer considers himself or herself at rest although he or she feels a force on him or her, the speed of flows of space-cells past him or her is not fixed, only the rate of their acceleration is. Hence an infinitely large number of rates of time-dilation and length-contraction of clocks and meter-sticks some distance away from the observer would come up as solutions. But only one of these solutions could possibly be a match with physical reality.

By contrast, a single solution with a definitive rate of time dilation and length contraction is yielded if the reference frame for which the solution has been found is that of an observer who feels no force on him or her. Right where he or she is, the speed of space cells is zero, and so is the rate of acceleration of the cells. This avoids any ambiguity, and so only one single solution with one single rate of length-contraction of meter-sticks and of time-dilation of clocks – both some distance away from the observer – presents itself for a given constellation of masses and their motions.

Hence, the (extended) relativity principle (according to which any observer who does not feel a force on him or her may consider himself or herself at rest) is absorbed into Einstein's field equation of General Relativity only from the moment on when a solution of this equation is sought after. It requires that a solution is to be found for an ordinary frame of reference (Cartesian, polar) in which an observer does not feel a force on him and her. Then reciprocity of relativistic effects between reference frames (in all of which the conservation of mass and momentum is guaranteed) is not certain *apriori*, but it yielded by a unique solution of Einstein's field equation, if the laws of mechanics and electromagnetism provide for it. That's exactly what we want.

So, if time **t'** in the reference frame **I'** is dilated from the perspective of the reference frame **I**, time **t** must also be dilated in **I** from the perspective of **I'**. Otherwise at least one of the two systems does not qualify as a system in which an observer is at rest (inertial system).

The famous twin-paradox thus shows that the traveling twin brother, who has returned from a roundtrip to a distant star and back, was not sitting in a reference frame entitled to consider itself as having been at rest all the time (presuming the twin brother who stayed at home *did* qualify as someone who was at rest all the time). For the unequal aging of the two twins was not reciprocal.

An observer at rest in a gravity field (mentioned by Einstein) is an analog to the traveling twin brother on his roundtrip to the stars and back. His or her unequal aging relative to someone who sits outside of the gravity field is absolute, and not reciprocal.

Any "liberalization" of the criteria for getting a "title" for being at rest (as argued for by A. Einstein in 1916, when he awarded the "title" of being at rest to his **K** and his **K'**, although neither of the two deserved it) would render General Relativity unscientific.

Hence, of the two cornerstones Einstein used for building General Relativity, one was insufficient for the job. Astonishingly or not, Einstein nevertheless obtained the correct result, namely, his field equation of General Relativity.

It should be noted that some authors think Einstein got back on the right track later on in his life. See P. Graneau/N. Graneau (2006), Chapter 8, p. 177: "*As a result, he* [Einstein] *defined this type of reference frame which we can call a free-fall frame as the only valid inertial frame in the theory of general relativity*."

Fortunately, the Schwarzschild- and Kerr-solutions of Einstein's field equation (and also the Reissner-Nordström solution) meet the required criterion: In each of the these cases, unprimed "coordinate time" and unprimed "coordinate space" are those of an observer at rest outside of the gravity field and hence free from forces. Moreover, within the realm of the Schwarzschild solution, an observer at rest in a *primed* system of coordinates whose origin is in free radial fall, that is, who travels along a geodesic, does, as expected, have a reciprocal experience of relativistic effects with respect to the observer in the unprimed frame (who is at rest outside of the gravity field in the unprimed system of coordinates). As regards the cosmic variant of the Schwarzschild solution, this was proved by A. Trupp (2024). Otherwise General Relativity would be inconsistent (contradictory).

To recapitulate: Einstein's field equation does, by itself, not say whether literally *all* observers are entitled to consider themselves as being at rest, or whether only those observers may do so who do not feel a force on them. In the former case, Einstein's field equation would be a tautology, and it would deliver a multitude of different solutions of which only one would possibly have physical significance. In order to avoid this, the extension of the relativity principle has to be limited to those observers who do not feel a force acting on them, that is, to observers in free (radial) fall. For reasons of symmetry, relativistic effects in different frames of reference of that kind must be reciprocal. Otherwise the (extended) relativity principle and thus General Relativity would be falsified. As a consequence, only those solutions of Einstein's field equation are sought after in which "coordinate-time" and "coordinate space" are those of an observer who does not feel a force on him or her. Thereby, and by nothing else, is the extended, but not overstretched relativity principle absorbed into Einstein's field equation.

2) Deriving the equivalence principle from Einstein's field equation

The principle of equivalence is indeed valid, although it is not a cornerstone of General Relativity or of Einstein's field equation, but a *consequence* of the latter. This shall be demonstrated in the following.

If we select one of the four coordinates which the general index **nu** stands for in (2), namely the coordinate **r**, our equation of the covariant divergence of the tensor $\mathbf{T^{\mu\ nu}}$ turns into:
(32)

$$\nabla_\mu T^{\mu r} = \delta_\mu T^{\mu r} + \Gamma^\mu_{\alpha\mu} T^{\alpha r} + \Gamma^r_{\alpha\mu} T^{\mu\alpha} = A^r = 0$$

The index **r** (equivalent to **nu** = 1) is what it is, namely the radial coordinate, and, different from all the other indices, does not run from 0 to 4. The vector $\mathbf{T^{\mu r}}$ refers to the interior of a test body, not to the vacuum in which it finds itself. Due to its smallness, the test body does not affect the metric tensor, which is shaped by the central spherical mass alone.

The equation is now the expression of a scalar, and no longer of a vector. It says: Whenever the ordinary divergence of the vector $\mathbf{T^{\mu r}}$ is different from zero and thus appears as a violation of the principle of conservation of momentum, it is because of the curvature of spacetime.

Let us multiply all sides of the equation with the non-zero scalar

$$\frac{dx^\mu dx^\alpha}{d\tau^2 T^{\mu\alpha}} \quad .$$

The term **x** is a generalized expression of coordinates, of which there are four ($\mathbf{x^0=t}$, $\mathbf{x^1=r}$, ...). We then get:
(33)

$$\frac{dx^\mu dx^\alpha}{d\tau^2 T^{\mu\alpha}} \nabla_\mu T^{\mu r} = \frac{dx^\mu dx^\alpha}{d\tau^2 T^{\mu\alpha}} \delta_\mu T^{\mu r} + \frac{dx^\mu dx^\alpha}{d\tau^2 T^{\mu\alpha}} \Gamma^\mu_{\alpha\mu} T^{\alpha r} + \Gamma^r_{\alpha\mu} \frac{dx^\mu dx^\alpha}{d\tau^2} = 0$$

On the other hand, we have a scalar equation that describes a geodesic [see L. Susskind/A. Cabannes (2023), Chapter 4, Equation 45, p. 158; instead of **r**, which would be chosen by L. Susskind /A. Cabanes (2023), **R** is used in **d²../d tau²** for reasons that shall not be explained, as the discrepancy is of no importance here]:
(34)

$$F_R = 0 \quad \Rightarrow \quad \frac{d^2R}{d\tau^2} + \Gamma^r_{\alpha\mu} \frac{dx^\mu dx^\alpha}{d\tau^2} = 0$$

It says: Whenever a test object, on which no force $\mathbf{F_R}$ is acting in a radial direction, exhibits an acceleration **d²R/dtau²** in the direction of **r**, it is because of the curvature of spacetime.

Again, the Christoffel-symbol, with its derivatives of elements of the metric tensor $\mathbf{g_{\mu\, nu}}$ [see (5)], is an expression of the curvature of spacetime. The term **R** is radial distance measured in numbers of stationary, radially oriented meter sticks laid end to end (and is not the Ricci scalar), **r** is radial distance measured in circumference of a circle around the center of the spherical mass, divided by 2 **pi**.

Then we can modify our equation (32) as follows, under the condition that no force $\mathbf{F_R}$ in a radial direction is acting on an object that would be responsible for its acceleration $\mathbf{d^2R/dtau^2}$:
(35)

$$(\frac{d^2R}{d\tau^2} \neq 0 \wedge F_R = 0) \Rightarrow \nabla_\mu T^{\mu 1} = \nabla_\mu T^{\mu r} = \frac{d\tau^2 T^{\mu\alpha}}{dx^\mu dx^\alpha}(\frac{d^2R}{d\tau^2} + \Gamma^r_{\alpha\mu}\frac{dx^\mu dx^\alpha}{d\tau^2}) = 0 \Rightarrow \frac{dv'_{space}}{d\tau} \neq 0$$

Time **tau** is the time of a stationary observer who sits close to the object that is undergoing an acceleration. Our equation now says: Whenever an object free from external forces nevertheless shows an acceleration and thus appears to violate the principle of conservation of momentum, it is so because the curvature of spacetime provides an acceleration of the space-cell in which the object is embedded. Any other interpretation would be contradictory: The object's acceleration WOULD violate the principle of conservation of momentum if the curvature of spacetime did *not* bring about an accelerating flow of space cells. It would then be unavoidable to acknowledge the presence of a force; however, this is what we had excluded. The presence of a force would be incompatible with the notion of a geodesic, where the absence of any force is essential. (35) therefore postulates that space itself is undergoing an acceleration in the reference frame of the local, stationary observer (whose time is **tau** and whose coordinate system is primed). In other words: *Accelerating flows of space are manifestations of the equivalence principle.*

It should be stressed that it is not the curvature of space alone, but the curvature of space and time, or simply of spacetime, that leads to an accelerating flow of space cells. For when writing out the Christoffel symbol with the help of the Schwarzschild metric, one finds that the only non-zero differential quotients summed up are $\mathbf{dg_{00}/dr}$ and $\mathbf{dg_{11}/dr}$. The former describes a change in time-dilation of stationary clocks with **r**, the latter a change in length-contraction of radially oriented, stationary meter sticks with **r**.

We are now about to realize: The "equivalence principle" is not, as is wrongly assumed in almost every article or book on Relativity, a foundation, that is, a starting point, of General Relativity. Instead, the principle of equivalence is DERIVED from Einstein's field equation and hence from the (extended) relativity principle. The relativity principle thus is the ONE AND ONLY foundation of General Relativity.

Given that gravitational "force" is nothing but accelerating flow of space, we find that weight does not exist in a strict sense. What we experience as weight is inertia which our body offers when electrostatic forces exerted by the surface of the earth (we stand on) act on the bottom side (soles) of our shoes. These forces accelerate our bodies in an upward direction, though not with respect to the surface of the earth, but with respect to an accelerating, anti-radial, that is, downward flow of space. Without these electrostatic forces, the accelerating downward flow of space would take our bodies along for the ride (what is does when we fall into a deep vertical shaft of a mine). In other words: a heavy mass is as large in magnitude as an inert mass, simply because heavy mass is nothing but inert mass.

3) More on flows of space as a manifestation of the equivalence principle

The accelerating flow of space-cells postulated by (35) becomes most evident in the cosmic variant of the Schwarzschild solution. We obtain this variant, if the tensor $\mathbf{T^{\mu\ nu}}$ (= $\mathbf{T_{ord\mu}}^{\mathbf{nu}}$) on the right-hand side of Einstein's field equation is set to zero, and a non-zero additional summand **-lambda** $\mathbf{g^{\mu\ nu}}$ is added on the left-hand side. The then-obtained variant of the Schwarzschild solution describes an expanding cosmic space (see above). A far-away observer, tethered to the Milky Way, experiences not only a time-dilation and a radial length-contraction (from the perspective of the Milky Way), but also a "force" that apparently wants to accelerate the tethered observer in a direction further away from the Milky Way. It therefore puts the tether under tension. All this happens just because space is steadily passing by that tethered observer at an accelerating rate. New space cells are steadily emerging between the Milky Way and the tethered, distant observer. There is simply no other mechanism available as an explanation (see A. Trupp, 2024), and this mechanism is widely accepted by the scientific community with respect to cosmic expansion. As long as the binding forces between molecules, and also the binding gravitational "forces" between stars, are stronger than the "force" generated by the accelerating space-cells that emerge between the stars, objects like solar systems and galaxies will not take part in the expansion of space.

The situation is not qualitatively different from ours when we stand on the surface of the earth. This is what (35) tells us. It is only the direction of an accelerating flow of space that is different: the flow is in an outward direction in the cosmic variant, and in an inward direction in the spherical-mass variant of the Schwarzschild solution.

Hence, in both variants of "Schwarzschild-observers", an inert mass can avoid a "going-along-for-the-ride" only if a real force comes to the aid. The real force encounters the inertia of the mass, and the inertia is mistaken for a "gravitational force".

Moreover, in the reference frame of an observer in free anti-radial fall (and thus at rest at the origin of his or her own frame of reference), there are tidal "forces" that act on him or her and try to stretch him or her ("spaghettification"), if he or she has a non-negligible radial extension (the "1000-mile-man", to use an expression by L. Susskind). These "forces" are the equivalent to the above-mentioned "force" on a far-away observer tethered to the Milky Way. Hence, tidal forces can be explained only as stated above, that is, by the fact that the extremities of an object which experience tidal forces are subject to an accelerating flow of space cells. This flow is brought about by the emergence of space cells.

The empirical impossibility of finding even the slightest quantitative difference between weight and inertia (principle of equivalence) doesn't thus come as a surprise. It is a corroboration of Einstein's field equation, and hence of the relativity principle from which Einstein's field equation is derived. In other words (and as a summary): Different from all textbooks on Relativity, the equivalence principle is not a starting point of General Relativity, but, as has been already said, is a CONSEQUENCE of Einstein's field equation and hence of the relativity principle.

VII. Relativistic effects as the results of space-flows

In this context, the following mathematical "fact" may come as a surprise: In both cases (cosmic variant and spherical-mass variant) of the Schwarzschild solution, the relativistic shortening of radially oriented, stationary meter sticks and the relativistic time-dilation of stationary clocks held by "Schwarzschild-observers" in gravity- or cosmic-expansion-fields are exactly the same in magnitude as the corresponding effects in flat Minkowski-spacetime. An observer in flat Minkowski spacetime (whose coordinate system has primed coordinates **t', x', y', z'**) who is in straight and unaccelerated motion in the unprimed system of coordinates **t, x,y,z** is subject to the same contraction of length of meter-sticks held in his hands and subject to the same dilation of time of a clock held in his hand as are the Schwarzschild observers, if his or her velocity is the same in magnitude as the escape velocity of the Schwarzschild-observers. In other words: Whenever the Minkowski-observer has a speed in the unprimed system of coordinates that is the same in magnitude as the escape velocity from the locations of the two "Schwarzschild-observers" in gravity- or cosmic-expansion-fields, the rates of time dilation and also of length contraction are the same. [To recall: The first Schwarzschild-observer stands on the surface of a spherical mass like a planet, and the second Schwarzschild-observer is tethered to the Milky Way as described above].

In mathematical terms: Since the local escape velocity from the surface (or from above the surface) of a spherical mass is $\mathbf{v^2_{esc}/c^2 = 2GM/rc^2 = r_s/r}$ both in Newtonian physics and in General Relativity (as derived from a geodesic on the basis of the outer Schwarzschild metric), we get
(36)

$$\frac{d\tau^2}{dt^2} = 1 - \frac{r_s}{r} = 1 - \frac{v'^{\,2}_{esc}(r)}{c^2} = 1 - \frac{v'^{\,2}_{space}(r)}{c^2}$$

$$\wedge \; \frac{ds^2}{dr^2} = (1 - \frac{r_s}{r})^{-1} = [1 - \frac{v'^{\,2}_{esc}(r)}{c^2}]^{-1} = [1 - \frac{v'^{\,2}_{space}(r)}{c^2}]^{-1}$$

Moreover, since the escape velocity of a galaxy in the cosmic variant of the Schwarzschild solution is $\mathbf{v_{esc}} = \mathbf{Hr}$, we get:
(37)

$$\frac{d\tau^2}{dt^2} = 1 - \frac{r^2H^2}{c^2} = 1 - \frac{v'^{\,2}_{esc}(r)}{c^2} = 1 - \frac{v'^{\,2}_{space}(r)}{c^2}$$

$$\wedge \quad \frac{ds^2}{dr^2} = (1 - \frac{r^2H^2}{c^2})^{-1} = [1 - \frac{v_{esc}^{/\,2}(r)}{c^2}]^{-1} = [1 - \frac{v_{space}^{/\,2}(r)}{c^2}]^{-1}$$

The equalities of time-dilation rates (**r**-dependent in both variants of the Schwarzschild metric, and $\mathbf{v_{esc}}$-dependent in the Minkowski metric) can hardly be the result of mere coincidences. The sameness can only be rooted in the undisputable fact that the local relative speed of a primed observer (whose coordinates are **x'**, **y'**, **z'**, **t'** or **tau**) with respect to space cells passing by him or her is the same in both cases (Minkowski and Schwarzschild).

One must, in addition, keep in mind that Special Relativity and General Relativity in the form of the Schwarzschild metric both have a contraction of meter-sticks and a dilation of time as their essential elements. Since Special Relativity is contained in General Relativity, the mechanisms responsible for these effects cannot substantially differ from each other.

We can thus set up the following statement:

In order to avoid inner contradictions, relativistic shortening of meter-sticks and relativistic dilation of time must each be the result of a flow of space-cells past the meter-stick and past the clock in the reference frame of an observer in which these effects occur. The only way to account for gravitational acceleration of objects is to assume that these objects are embedded in an accelerating flow of space-cells. Any attempt to disrupt this "going-along-for-the ride" needs an external force on the object.

Finally, one must not forget that the concept of space-flows is indispensable for explaining why Einstein's field equation provides an infinite number of solutions for a given constellation of masses and their motions in case literally all observers were entitled to consider themselves at rest (see above), but do *not* do so, if only those observers are allowed to consider themselves at rest who do not feel any force on them. Only the concept of space-flows explains why, in the latter case, merely one unique solution is provided (see above).

VIII. Einstein on flows of space

It has to be stressed that a flow of space is *no* re-introduction of a *classical* ether. (35) postulates that space has the capacity of being in a state of acceleration, and that objects embedded in accelerating space-cells go along for the ride. (36) and (37) demonstrate that a speed can be described to space cells, but this speed is not absolute. Instead, it depends on the reference-frame chosen. An observer in free anti-radial fall who has a speed that differs in absolute amount from that of escape velocity, that is, from the speed of free fall from afar (e.g. by exceeding that speed), is entitled to consider the space-cell around him or her as being at rest. By contrast, another observer, for instance, an observer at rest outside of the gravity field (for whom space cells are moving as fast as a freely falling observer from afar would), sees space-cells pass by the falling observer at (constant) non-zero relative speed.

As space thus lacks of a definitive speed, our equations (35), (36) and (37) are descriptions not of a *classical* ether, but of an "ether" of General Relativity. A. Einstein (1922) gave a description of this "ether" in his famous lecture at the University of Leiden in 1920:

"*More careful reflection teaches us however, that the special theory of relativity does not compel us to deny ether. We may assume the existence of an ether; only we must give up ascribing a definite state of motion to it, ... To deny the ether is ultimately to assume that empty space has no physical qualities whatever. The fundamental facts of mechanics do not harmonize with this view. ...what is essential is merely that besides observable objects, another thing, which is not perceptible, must be looked upon as real, to enable acceleration or rotation to be looked upon as something real*."

This "other thing" is flowing space. Einstein expanded on this as late as in 1952, when he eventually introduced the concept of flowing spaces into his physics. As is the case with flows of electromagnetic energy (which are made "visible" by the Poynting-vector), a flow of space, too, is frame-dependent, so that there can be an infinite number of flows of space in one and the same volume element. In A. Einstein's words [A. Einstein (1961), Appendix V – supplemented by Einstein in 1952 –, pp. 138, 139]:

"*Before one has become aware of this complication, space appears as an unbounded medium or container in which material objects swim around. But it must now be remembered that that there is an infinite number of spaces, which are in motion with respect to each other. The concept of space as something existing objectively and independent of things belongs to pre-scientific thought, but not so the idea of the existence of an infinite number of spaces in motion relatively to each other. This latter idea is indeed logically unavoidable, but is far from having played a considerable role even in scientific thought.*"

The identification of accelerating flows of space-volumes ("cells") as the cause of what we call "weight of stationary, supported bodies in a gravitational field" is not a concept that is new to Relativity in principle. See for instance H. Reichenbach (1958), Chapter III, § 36, pp. 225/226:

"*Generally speaking, we can transform away gravitational fields only in infinitesimal regions. Let us consider for example the radial field of the earth (Fig. 41). If we let a rigid system of cells (the dotted lines of the figure) move in the direction of arrow **b** with an acceleration $g = 981\ cm/sec^2$, the earth field will be transformed away in a cell **a** but not in any of the others. ... We may therefore say that any gravitational field can always be transformed away in any given region, but not in all regions at the same time by the same transformation.* "

See also W. Pauli (1921/1981), paragraph 51, p. 145:

"*In short, in an infinitely small world region every gravitational field can be transformed away*.".

Contrary to the two cited statements, a transforming away of a gravitational "force" by flows of space is not restricted to infinitesimal regions. It can be done over large regions of space as well. One simply has to acknowledge that space volumes are capable of emerging apparently out of nothingness. Such is, for instance, undisputedly the case in the cosmic variant of the Schwarzschild solution and its expanding cosmic space. One also has to acknowledge that space volumes are capable of vanishing apparently into nothingness. Such is, for instance, the case with respect to space volumes that steadily enter planet earth from outside in order to vanish into nothingness in its interior.

IX. The equivalence principle inside spherical masses

A special situation presents itself in the interior of spherical masses. In case the freely falling, radially extended observer continues his or her free fall past the surface of the spherical mass through a shaft in its interior, tidal "forces" would now try to crush him or her. The anti-radial, that is, pushing "force" on the *trailing* extremity of a freely falling object is stronger than that on the *heading* extremity. This is because of the following: Although the velocity of free fall increases with depth, the rate of speed-increase, that is, the acceleration of the freely falling observer, *decreases* with depth.

Again, it turns out that the rate of time-dilation of a stationary clock in the shaft is the same as the rate of time-dilation of a clock that travels in uniform and straight motion in flat Minkowski-space at a velocity that is identical in amount to that of free fall (which started far away from the spherical mass in empty space). However, the inner Schwarzschild solution that yields this result also yields the following result: The described match does not apply to the relativistic shortening of meter sticks. Instead, the rate of length-contraction (a stationary meter-stick in the shaft is subject to) is the same as that of a meter-stick that is traveling in Minkowski-space at a *lower* speed than that of free fall in the shaft.

In order to realize this strange feature, we (once more) consider the inner Schwarzschild solution. It reads (**r_s** is the Schwarzschild radius, **R_0** is the radius of the spherical mass):
(38)

$$c^2 d\tau^2 = \frac{1}{4}\left(3\sqrt{1-\frac{r_s}{R_0}} - \sqrt{1-\frac{r^2 r_s}{R_0^3}}\right)^2 c^2 dt^2 - \frac{1}{1-\frac{r^2 r_s}{R_0^3}} dr^2 - r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2$$

A clock at rest at the center of the spherical mass (**r=0**) experiences a noticeable dilation of time, that is **$dtau^2/dt^2= [3(1-r_s/R_0)^{0.5}-1]^2/4 < 1$** (given **$r_s>0$**). In contrast, a meter-stick at rest at the center does not experience any contraction in length at all (**ds^2/dr^2** =1). Hence, the corresponding velocity of the meter-stick in Minkowski-space is zero, and is thus not the velocity of a free fall from afar (identical in amount with the local escape velocity). As regards the contraction of stationary meter sticks, it is the speed of the *net* (=resulting) flow of space in a radial or anti-radial direction that determines its extent. As regards the behaviour of stationary clocks, any direction of a net flow of space past the clock has to be as good as any other.

The only explanation of that disparity that is not self-contradictory is the following: Near the center of the spherical mass, there is a (net) flow of space-cells in the direction of a fourth spatial dimension **w**, but a zero net flow in any other direction. This is how flowing space volumes (which are to be distinguished from possible flows of vacuum energy) leave three-dimensional space in the interior of the spherical mass. In detail: Two separate flows of space overlap and superpose each other in the interior of a spherical mass. One flow (primary flow) is the expected flow whose speed is that of a freely falling test body (who started his or her fall far away from the spherical mass atzero speed). The other flow (secondary flow) is a flow in the opposite direction. It has its maximum speed at the center of the spherical mass (at **r=0**), where it is equal in magnitude to the primary flow. It has its minimum speed at $\mathbf{r=R_0}$, that is, when reaching the surface, where its speed is zero. The existence of that second flow of space is also a consequence of the relativity principle; however, this shall not be shown here.

In mathematical terms [following from (36)]:
(39)

$$\frac{ds^2}{dt^2} = [1 - \frac{v'^2_{space_R}}{c^2}]^{-1} = 1 \Rightarrow v'_{space_R} = 0$$

$$\frac{d\tau^2}{dt^2} = 1 - \frac{v'^2_{space_w}}{c^2} < 1 \Rightarrow v'_{space_w} \neq 0$$

For a supported test mass at rest in the shaft, it is the rate of change of the primary flow only that is responsible for the "weight" of the mass. A flow of space does not exert a "force" on a supported mass at rest if that flow of space is constant in speed. Obviously, thesecond flow (counter flow) does also not exert a "force" on a supported, stationary test mass. Again, this phenomenon is explained by the relativity principle. But this explanation shall not be presented here.

A reconsideration of Kaluza's theory will bring us back to the topic of a fourth spatial dimension.

X. The behaviour of electric charge in a gravity field

The ubiquitous (and not merely infinitesimal) "transforming-away" of any gravity is confirmed when considering electric charges in a gravitational field (outside of a spherical mass).

A charge accelerated by an external electric field outside of a gravity field does not radiate in a usual sense (there is no frequency that could be ascribed to the accelerated charge as long as its acceleration continues). What it does is: It is subject to its own deformed electrostatic field that reacts to its source, the charge, and exerts a back-force on it. This is the only way how the inert mass of the energy of the electrostatic field in flat spacetime (Minkowsi-space) can offer mechanical resistance to its being accelerated. [See C. de Almeida/A. Saa (2006), p. 154: "*As we will see, uniformly accelerated observers are able, in principle, to detect electromagnetic radiation from an inertial charge. These observations are enough to solve the paradoxes posed ...*".]

A stationary, supported charge in a gravitational field does the same. It displays the same deformed electric field for a bystander as does an accelerated charge in Minkowski-space for a co-accelerated observer [see A. K. Singal (1997), p. 1389, with a figure showing field lines suggested by J.A. Wheeler]. There is a simple reason why this has to be so: If this were not the case, the "weight" of the energy of the electrostatic field of the charge could not be communicated to the surface of the earth. That energy, which is equivalent to mass, would thus be exempt from the influence of gravity – which we can rule out.

Conversely, a charge in free anti-radial fall in a gravity field cannot display such a deformed electrostatic field. It it did, it would, because of the backforce exerted on the charge by its own electrostatic field, make the charge increase its speed of free fall at a lower rate in comparison with electrically neutral masses in free fall. This can also be ruled out, since all masses share the same rate of speed-increase when in free fall. [See A. Shariati, M. Khorrami (1999), p. 439, who correctly state that a freely falling electric charge in a gravitational field does "*not radiate in the sense that no extra force is needed to maintain their world-line the same as that of a neutral particle*".] The electrostatic field of a freely falling charge is thus indistinguishable from the electrostatic field of an unaccelerated charge in Minkowsi-space, at least if we restrict our view to small adjacent regions of space that surround the charge. Then we have all we need to be entitled to say that space cells themselves are in accelerating motions.

Since both stationary (supported) charges and also charges in free fall could be arranged to exist in large numbers on or above the surface, that is, in space around planet earth, the concept according to which a transforming-away of a gravitational field by means of flowing, accelerating space is restricted to infinitesimal regions is revealed to be untenable. Instead, accelerating flow of space that replaces a gravitational force is a ubiquitous phenomenon. This is also what (35) postulates.

XI. Kaluza's theory of a unification of electromagnetism and gravitation reconsidered

1) The addition of the tensor elements $\mathbf{T^{40}}$, $\mathbf{T^{41}}$, $\mathbf{T^{42}}$, $\mathbf{T^{43}}$, $\mathbf{T^{44}}$ instead of $\mathbf{g^{40}}$, $\mathbf{g^{41}}$, $\mathbf{g^{42}}$, $\mathbf{g^{43}}$, $\mathbf{g^{44}}$

Back to the relativity principle that forms the only basis of Relativity. The relativity principle requires that, in addition to the conservation of mass and momentum, any observer at rest must also find the principle of conservation of electric charge to be valid. For this simple reason, the $\mathbf{T^{\mu\,nu}}$ tensor, which we so far have treated as a symmetrical 4 x 4- tensor, can and *must* be expanded to a symmetrical 5 x 5-tensor. The new element $\mathbf{T^{40}}$ (and also $\mathbf{T^{04}}$) is charge density **sigma** (in units of Coulomb/m³). The new element $\mathbf{T^{41}}$ (and also $\mathbf{T^{14}}$) is charge flux in the **x**-direction [in units of Coulomb/(m² sec)]. The new element $\mathbf{T^{42}}$ (and also $\mathbf{T^{24}}$) is charge flux in the **y**-direction [in units of Coulomb/(m² sec)]. The new element $\mathbf{T^{43}}$ (and also $\mathbf{T^{34}}$) is charge flux in the **z**-direction [in units of Coulomb/(m² sec)]. Finally, the new (diagonal) element $\mathbf{T^{44}}$ is charge flux in the **w**-direction, if any [in units of Coulomb/(m² sec)], with **w** being a fourth spatial dimension. Electric charge can be conceive of as being smeared out and thus constituting a homogeneous "charge paste" [as was suggested even for electric *dipoles* by L. Eyges (1980), Chapter 10.6, p. 162].

As did Th. Kaluza in 1921, we, too, set up the restriction (side condition) that no parameter depends on the fourth spatial dimension. As a consequence, differentials containing **dw** must not be integrated. We nevertheless allow (as did Kaluza) **dw** to be differentially small, that is, different from zero, and do not set **dw=0**. [A similar restriction is tacitly contained in the unaltered version of Einstein's equation as well, see A. Trupp (2022), since flows of mass and momentum are presumed – and not *proved* – to be confined to three spatial dimensions.] Einstein's field equation is still valid, even with **µ** and **nu** in all tensors now running from 0 to 4, and these indices being **t**,**x**,**y**,**z**,**w** (in Cartesian coordinates).

Different from what Kaluza did, we do not begin our chain of steps by expanding the metric tensor $\mathbf{g}^{\mu\ nu}$ that appears on the left-hand side of Einstein's field equation, but by expanding the energy-momentum tensor $\mathbf{T}^{\mu\ nu}$ that appears on the right-hand side. That makes a huge difference. The latter is zero in vacuum, the former is not. Moreover, the extension of the energy-momentum tensor $\mathbf{T}^{\mu\ nu}$ is necessitated by the relativity principle. It is long overdue, since all conservation principles – and not only the principles of conservation of mass and of momentum – must be observed in any rest frame if the (extended) relativity principle is to hold true. The effect which this extension is having on the $\mathbf{g}^{\mu\ nu}$-tensor must therefore be found by *solving* the extended, that is, 5 x 5- version of Einstein's field equation on the basis of the meanings attributed to the new tensor elements $\mathbf{T}^{40}$, $\mathbf{T}^{41}$, $\mathbf{T}^{42}$, $\mathbf{T}^{43}$, $\mathbf{T}^{44}$, that is, on the basis of a complete knowledge about all the elements of the 5 x 5-tensor $\mathbf{T}^{\mu\ nu}$. But this is not what Kaluza provided us with!

[Like the Schwarzschild solution (which is based on $\mathbf{T}^{00}$ being mass density, and all the other elements of the 4 x4-tensor $\mathbf{T}^{\mu\ nu}$ being zero), this sought-after solution must present itself in the form of a "line-element".]

2) Extracting Maxwell's equations from Einstein's expanded field equation

It does not come as a surprise: Maxwell's equations of electricity and magnetism are contained in Einstein's correctly expanded field equation, but not due to the added elements alone, but due to old and new elements combined.

As regards the old elements, they yield the law of the invariance of the speed of light (see above). In order to recognize the role of the invariance of the speed of light for extracting Maxwell's laws from Einstein's correctly expanded field equation, we start from what we obtained above, namely:

(40)

$$v_{light}^2 = c^2 = \frac{1}{\varepsilon_0 \mu_0}$$

We start from even more premises: We know that a front of a combination of two fields moves forward at a constant speed **c**, with the directions of each of the two fields being perpendicular both to the direction of motion of the front and to the other field. In other words: We know that the phenomenon "light" is composed of two time-varying fields which are perpendicular to each other and to the direction of motion of the front. We take this as implicitly contained in $\mathbf{v^2_{light}} = \mathbf{c^2}$. We also know that one field, which we call **E**, exerts a force **F** both on an electric charge **q** at rest (**F=qE**) and also on an electric charge in motion, whereas the other field, which we call **H** or **B**, exerts force **F** only a an electric charge **q** in motion (**F=qvB**, with the speed **v** and the field **B** being perpendicular to each other), and not on an electric charge at rest.

Multiplying both sides by the vector **E**, and recalling what we know about "light", gives:
(41)

$$\vec{E} = \vec{E}\, v_{light}^2\, \varepsilon_0\mu_0 = \mu_0(\vec{v}_{light} \times \varepsilon_0\vec{E}) \times \vec{v}_{light} = a(v_{light})\mu_0\vec{H} \times \vec{v}_{light} = a(v_{light})\vec{B} \times \vec{v}_{light}$$

The term "**a**" is a still unknown, dimensionless factor that depends on the numerical value of the speed of light. $\mathbf{B} = \mu_0\mathbf{H}$ is a definition of **B**.

If a factor **q** (charge in Coulomb) is added, the equation gives the force on a charge **q**:
(42)

$$\vec{F} = q\vec{E} = qa(\vec{B} \times \vec{v}_{light}) = q(\vec{B} \times \vec{v}_{light})$$

The velocity $\mathbf{v_{light}}$ now is the speed of the charge. Since a magnetic field **B** or **H** is defined as exerting a force $\mathbf{F=qvB=q\mu_0vH}$ on a moving charge **q** if **v** and **B** are perpendicular to each other (see above), the factor **a** (as a function of the numerical value of $\mathbf{v_{light}}$) in (41) must be equal to unity. One should note: If $\mathbf{v_{light}}$ were not a constant, (42) would be self-contradictory, given **E** does not depend on **v**. and given **B** is not indirectly proportional to **v**.

[Moreover, the electric force on a charge traveling at speed **c** is hence the same in magnitude as the Lorentz force acting on that charge caused by a magnetic field that is als present. This appears to be strange, as the strength of the electric field does not appear to have any influence on what the strength of the magnetic field might be. As early as in 1856, W. Weber and R. Kohlrausch labeled the speed **c** as the "critical" speed at which the magnetic force – later called "Lorentz force" – on a moving charge was equal to the electrostatic force. See R.W. Pohl (1975), Chapter 9. § 3, p. 79. We will come back to this conundrum later on.]

We hence get (as a consequence of the invariance of the speed of light in vacuum):
(43)

$$\vec{E} = \vec{B} \times \vec{v}_{light} = \vec{B} \times \vec{c}$$

When now imagining [as did R.P. Feynman (1965), Chapter 18-4, pp. 18-5 to 18-8] that an infinite, flat sheet evenly charged with electricity of a single sign is suddenly shifted tangentially over a limited time after which the shifting comes to an end (whereas a second sheet with charge of the opposite sign that is in sliding, but insulated contact with the former sheet does not take part in this tangential motion), we can assume that a block of homogenous **B**- or **H**-field-strength is created. Knowing all of the above, the block must emanate from the sheet and must move in the direction of the plane's normal into infinity. (43) thus says that the motion of the **B-** or **H**-block is accompanied by the creation of an electric field **E** (which also forms a homogeneous, moving block), whose direction is perpendicular both to the magnetic field **H** and to the direction of motion of the heading face of the block.

(43) can be converted into Faraday's law by forming the curl of its very left and its very right side:
(44)

$$\vec{\nabla} \times \vec{E} = \vec{\nabla} \times (\vec{B} \times \vec{c}) = -\frac{\delta \vec{B}}{\delta t}$$

The reason why the curl of the cross product of **B** and **c** is equal to **-dB/dt** is the following: In the infinitesimally deep border region of the traveling **B**-block (heading face), the field **E** is equal in magnitude (not in direction) to its own curl for geometrical reasons. This must then be true for the cross product of **B** and **c** as well; it, too, must be identical in magnitude (not in direction) with its own curl. Right there in the border region, the cross product of the vectors **B** and **c** is, in turn, equal in magnitude to **-dB/dt**, that is, to the change in magnetic flux with time, for simple geometrical reasons. As the direction of the curl of the cross product of **B** and **c** must be perpendicular to **E**, it must (as does the curl of **E**) either point in the direction of **B** or of **c**. For geometrical reasons, one finds it must point in the direction of **B**. Thus the curl of the cross product of **B** and **c** must be equal to the vector **-dB/dt** both in magnitude and direction. (The negative sign in front of **dB** is due to a convention on which is which direction of rotation.)

Similarly, our equation
(45)

$$\vec{c} \times \varepsilon_0 \vec{E} = \vec{H} = \frac{\vec{B}}{\mu_0}$$

can be converted into the second half of Ampere's law by forming the curl of both sides of (45):
(46)

$$\vec{\nabla} \times \mu_0 \vec{H} = \vec{\nabla} \times \vec{B} = \vec{\nabla} \times \mu_0(\vec{c} \times \varepsilon_0 \vec{E}) = \mu_0 \varepsilon_0 \frac{\delta \vec{E}}{\delta t}$$

To elucidate: Given that the block of homogeneous **E** (which is generated by the change in **B**), too, is moving in the direction of the plane's normal, this motion of the **E**-block results in a steady "refreshing" of the **H**-field according to (45)**.** In the infinitesimally deep border region (the heading face of the block), the field **H** or **B** is equal in magnitude to its own curl for geometrical reasons. Moreover, in the border region, the cross product of the vectors **c** and

E is equal in magnitude to **dE/dt**, that is, to the change in electric flux with time. As the direction of the curl of the cross product of **c** and **E** must be perpendicular to **H**, it must (as does the curl of **H**) either point in the direction of **E** or of **c** according to (45). For geometrical reasons, it must point in the direction of **E**. Thus the curl of the cross product of **c** and **E** must be equal to the vector **dE/dt** both in magnitude and direction.

(44) can be reformulated as (with $\mathbf{a_1}$ and $\mathbf{a_2}$ being infinitesimally small numbers):
(47)

$$\delta t\ (\vec{\nabla} \times \vec{E}) = a_1\ (\vec{\nabla} \times \vec{E}) = \delta\vec{B} = -a_2\vec{B}$$

Since the divergence of a curl is always zero, (47) thus yields:
(48)

$$\vec{\nabla} \cdot \vec{B} = \vec{\nabla} \cdot [\frac{a_1}{a_2}\ (\vec{\nabla} \times \vec{E})] = 0$$

This is Maxwell's third equation.

We thus realize that the principle of the invariance of the speed of light yields Faraday's law, as well as the second part of Ampere's law (with the flux of electric charge – as another source of the magnetic field besides the displacement current **dE/dt** – still missing), and also Maxwell's third equation. For the validity of our equation $\mathbf{v^2_{light} = c^2}$ (from which all of this was derived), it makes no difference whether or not the two sheets are in motion in the direction of their normals.

The law of the invariance of the speed of light is a result that we obtained prior to the expansion of the tensor $\mathbf{T^{\mu\ nu}}$. Let us now turn our attention to the role of the new tensor elements $\mathbf{T^{40}}$, $\mathbf{T^{41}}$, $\mathbf{T^{42}}$, $\mathbf{T^{43}}$, $\mathbf{T^{44}}$, and what role they play in (46). Equation (46) is not yet complete for the following reason: It implicitly says that the divergence of the vector **E** is always zero. For (46) can be re-written as ($\mathbf{b_1}$ and $\mathbf{b_2}$ are infinitesimally small numbers):
(49)

$$\mu_0\varepsilon_0\delta\vec{E} = \mu_0\varepsilon_0 b_1\vec{E} = \delta t\ (\vec{\nabla} \times \vec{B}) = b_2\ (\vec{\nabla} \times \vec{B})$$

Since the divergence of a curl is always zero, (49) thus yields:
(50)

$$\vec{\nabla} \cdot \vec{E} = \vec{\nabla} \cdot [\frac{b_2}{\mu_0\varepsilon_0 b_1}(\vec{\nabla} \times \vec{B})] = 0$$

However, there are electric charges charges in this world. They are brought into the picture by the new tensor elements $\mathbf{T^{40}}$, $\mathbf{T^{41}}$, $\mathbf{T^{42}}$, $\mathbf{T^{43}}$, $\mathbf{T^{44}}$. Saying that a charge density exists means that electric field lines exist that originate or terminate in charges. Charges do not come without them; this is implied in their definition. Therefore, in physical reality, electric field lines do not come in loops only. This is why some work has do be done on (46). The following

modification of (46) presents itself as the only means of always getting a zero-divergence of both sides of (46) *without* requiring that all electric field lines always come in loops (with **j** being charge-flux in Coulomb per second and per m²):
(51)

$$\vec{\nabla} \times \vec{B} = \frac{\vec{j}}{\varepsilon_0} + \mu_0 \varepsilon_0 \frac{\delta \vec{E}}{\delta t}$$

The certainty of:
(52)

$$\vec{\nabla} \cdot \left(\frac{\vec{j}}{\varepsilon_0} + \frac{\delta \vec{E}}{\delta t}\right) = 0$$

is again provided by the fact that the divergence of a curl [and hence also that in (51)] is always zero, and, of course, by the additional fact that no inner contradictions arise from the modification of (46) when adding a summand **j** in (51).

(51) represents the complete equation of Faraday's law.

We can go even further. The principle of conservation of charge, which is expressed through the fact that the covariant divergence of the new vector $\mathbf{T}^{40}$, $\mathbf{T}^{41}$, $\mathbf{T}^{42}$, $\mathbf{T}^{43}$, $\mathbf{T}^{44}$ is zero, can be given the following form (**rho** is charge density, **j** is charge flux):
(53)

$$\vec{\nabla} \cdot \vec{j} = -\frac{\delta \rho}{\delta t}$$

Using this in (52) gives:
(54)

$$\vec{\nabla} \cdot \left(\frac{\vec{j}}{\varepsilon_0} + \frac{\delta \vec{E}}{\delta t}\right) = \vec{\nabla} \cdot \frac{\vec{j}}{\varepsilon_0} + \vec{\nabla} \cdot \frac{\delta \vec{E}}{\delta t} = -\frac{\delta \rho}{\varepsilon_0 \delta t} + \vec{\nabla} \cdot \frac{\delta \vec{E}}{\delta t} = 0$$

Or, after multiplying all sides of (54) by $\mathbf{epsilon_0}$:
(55)

$$-\frac{\delta \rho}{\delta t} + \varepsilon_0 \vec{\nabla} \cdot \frac{\delta \vec{E}}{\delta t} = -\frac{\delta \rho}{\delta t} + \varepsilon_0 \frac{\delta}{\delta t} \vec{\nabla} \cdot \vec{E} = 0$$

The middle part of (55) is obtained by changing the order of partial differentiation.

Due to the concept of "charge" as a thing which is the origin or end-point of **E**-field-lines (whose number is not necessarily proportional to the amount charge), the charge-density **rho**

must be zero whenever the divergence of **E** is zero. Therefore, when integrating (55) over **t**, the constant of integration must be zero. An integration of (55) thus yields:
(56)

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

This is Gauss' law. That law and also the completion of Faraday's law both required additional tensor elements, and could not have been extracted from Einstein's original field equation that has 4 x 4 tensors only.

We realize (and repeat): An expansion of the 4 x 4 symmetrical tensors in Einstein's field equation to 5 x 5 symmetrical tensors in the way described above comes as a necessity as soon as one recognizes that the covariant divergence of the tensor $\mathbf{T}^{\mu\ nu}$ on the right-hand side of Einstein's field equation is an expression of the conservation laws we know of. Apart from the law of conservation of mass and momentum, there is also the law of conservation of electric charge. This law is missing in the 4 x 4 version of the tensors in Einstein's field equation. After an expansion of the $\mathbf{T}^{\mu\ nu}$ tensor (and hence of all tensors) to 5 x 5, all of Maxwell's equations of electromagnetism can be extracted from Einstein's field equation.

3) Why an electromagnetic field cannot be "transformed way"

An *exact* solution of Einstein's extended 5 x 5- field equation for a (non-spinning) spherical mass with evenly distributed charge in its interior is still to be found. Kaluza's proposal for the 5 x 5- metric tensor $\mathbf{g}_{\mu\ nu}$ was produced out of thin air, and cannot qualify as the desired solution. Kaluza simply contracted the 4 x 4 electromagnetic Maxwell tensor to a single vector with four components (elements). These elements were then identified with the elements $\mathbf{g}^{40}$, $\mathbf{g}^{41}$, $\mathbf{g}^{42}$, $\mathbf{g}^{43}$ of the (new) symmetrical 5 x 5 metric tensor $\mathbf{g}^{\mu\ nu}$ as a simple guess. The tensor element $\mathbf{g}^{44}$, for which no contents was provided, was seen as irrelevant. But the whole thing wasn't serious business. In order to do it correctly, the search for a solution, that is the determining of all the elements of the symmetrical 5 x 5 tensor $\mathbf{g}^{\mu\ nu}$ or its inverse, has to start from what we know about the elements of the tensor $\mathbf{T}^{\mu\ nu}$.

Even before we will have found a solution of Einstein's expanded 5 x 5 field equation, we realize that the ordinary electric field, unlike the gravitational field, cannot be "transformed away". To realize this, one may return to (35):

$$(\frac{d^2R}{d\tau^2} \neq 0 \wedge F_R = 0) \Rightarrow \nabla_\mu T^{\mu 1} = \nabla_\mu T^{\mu r} = \frac{d\tau^2 T^{\mu\alpha}}{dx^\mu dx^\alpha}(\frac{d^2R}{d\tau^2} + \Gamma^r_{\alpha\mu}\frac{dx^\mu dx^\alpha}{d\tau^2}) = 0 \Rightarrow \frac{dv'_{space}}{d\tau} \neq 0$$

In order for a force **F** in the direction of **R** to be transformed away (that is, to be zero), the local acceleration of a test-object must be equal (in absolute amount) to the product of the Christoffel symbol and the differential quotient next the Christoffel symbol. Although we do not yet know how exactly the metric tensor is affected by the presence of charge, we can surely exclude that the modified metric will enable **$d^2R/dtau^2$** in the equation to be the acceleration of a charge in an electric field. This is for a simple reason: We know by experience that an electric charge accelaterated by an electric field radiates (in the sense described above). But this is incompatible with a charge who simply follows a geodesic, and whose acceleration is therefore only the result of a going-along-for-the-ride which the space-cell around it offers.

There is even another objection. If the electric force could be transformed away, Coulomb's law would still have to retain the quality of being reproducable from spacetime-geometry. That is to say: In the same way in which Newton's law of gravitation can be reproduced from the Schwarzschild solution, Coulomb's law would have to be be reproducable from a line-element that, in otherwise flat spacetime, would look a lot like the outer Schwarzschild solution, with only **GM** substituted by **kQ**. But the curvature of spacetime would exist for electric charge only, not for test-objects that are electrically neutral. This would be incompatible with General Relativity. In General Relativity, the metric tensor may depend on the reference frame chosen, but not on the material of the test-object.

(Since the electric force does not qualify for being a non-force, the covariant divergence of mass and momentum is zero only if mass and momentum of the electric field, too, is given consideration.)

Even if there were singular cases in which the Poynting vector would tell us that electromagnetic energy apparently emerges out of nothingness or vanishes into nothingness, these phenomena could not, as long as charge accelerated by the electromagnetic field radiates, be "transformed away". Only by a resorting to a fourth spatial dimension into which any mass might disappear or from which it might emerge (and not by blaming it on the curvature of spacetime) could such a phenomenon be accounted for. An escape into or a coming from the fourth spatial dimension is given an expression by the tensor element **T^{40}** (see below).

XII. Kaluza's modified theory applied to the Trouton-Noble paradox

1) The Trouton-Noble paradox

The so-called "Trouton-Noble paradox" has so far been unsolved, as all attempts made to solve it have not been fully satisfactory. One of its variants is the following: Two electric point-charges of the same sign and magnitude shall be in rapid, straight and uniform motion in the horizontal **x**-direction in flat spacetime (Minkowski-space). A special moment in time shall be scrutinized in which one point-charge is at **x**=+1, **y**=+1, **z**=0, whereas the other charge is at **x**=-1, **y**=-1, **z**=0. The electrostatic field generated by each of the two point-charges is not spherically symmetrical, but is (relativistically) compressed in the direction of motion. Nevertheless, all field lines are straight and pass right through the point-charge [see E.M. Purcell (1985/2011), Chapter 5.6., Fig. 5.14, p. 186]. Consequently, in the reference frame **x**,**y**,**z** of an observer at rest, each point-charge is subject to a repulsive electrostatic force from the other charge directed strictly along the connecting line between the two charges.

However, each charge is also subject to a *magnetic* field generated by the other charge. Therefore each charge is subject to a Lorentz-force strictly pointing in the positive or negative **y**-direction. Let the paths of the two point charges be guided by rails, so that the Lorentz-force does not result in any acceleration of the point charges. Nevertheless, the Lorentz force leads to pressure on the rails.

In the rest frame of the two point-charges, the electrostatic force exists (directed along the connecting line between the two point-charges); the Lorentz-force, however, does not exist. This seems to violate the relativity principle.

Attempts have been made to overcome this dilemma by distinguishing between the direction of a force on an object on the one hand, and the direction of the acceleration brought about by that force on the other hand [R.C. Tolman (1911), P.S. Epstein (1911)]. But this is no issue here, since the two charges are supposed to be guided by rails, and no acceleration perpendicular to the **x**-direction is brought about. Other attempts consisted in resorting to a torque that was imagined to be generated by molecular forces in the interior of the rod connecting the two charges. That imagined torque was postulated to be exactly cancelled by the Lorentz-force [see M. von Laue (1911)]. However, in our thought experiment, no connecting rod exists between the two charges. We may simply imagine that this connecting rod was removed a moment ago, so that the velocity of the two point charges has not yet changed despite the action of the repulsive electrostatic forces. [For an overview on the long-standing discussion, see J. Franklin (2006).]

2) The relativstic electric field " $\mathbf{E}_{\mathbf{rel}} = \mathbf{B} \times \mathbf{v}$ " of a moving charge as the solution of the paradox

The only way out of this dilemma is the following: Each moving charge does not only produce a magnetic field, but also a relativistic *electric* field (similar to what a bar-magnet in motion does). This relativistic field exactly cancels the Lorentz-force that acts on a moving charge **q** [the Lorentz-force being $\mathbf{F}_{\mathbf{lorentz}} = \mathbf{q}\ (\mathbf{v} \times \mathbf{B})$].

In order to realize this, we consider the curl of a cross product of two vectors **B** and **v**. This gives:
(57)

$$\vec{\nabla} \times (\vec{B} \times \vec{v}) = (\vec{v} \cdot \vec{\nabla})\vec{B} - (\vec{B} \cdot \vec{\nabla})\vec{v} + \vec{B}(\vec{\nabla} \cdot \vec{v}) - \vec{v}(\vec{\nabla} \cdot \vec{B}) = (\vec{v} \cdot \vec{\nabla})\vec{B} - (\vec{B} \cdot \vec{\nabla})\vec{v}$$

$$= [(\vec{v} \cdot \vec{\nabla}B_x)\vec{e}_x + (\vec{v} \cdot \vec{\nabla}B_y)\vec{e}_y + (\vec{v} \cdot \vec{\nabla}B_z)\vec{e}_z] - [(\vec{B} \cdot \vec{\nabla}v_x)\vec{e}_x + (\vec{B} \cdot \vec{\nabla}v_y)\vec{e}_y + (\vec{B} \cdot \vec{\nabla}v_z)\vec{e}_z]$$

We now assume that **v** is the constant speed of a charge in motion. That motion shall be strictly in the positive **x**-direction. **B** shall be the magnetic field generated by that charge in motion. The gradient of each component of **v** is zero. Hence, the divergence both of **v** and of **B** is zero. The equation thus turns into:
(58)

$$\vec{\nabla} \times (\vec{B} \times \vec{v}) = (\vec{v} \cdot \vec{\nabla})\vec{B}$$

$$= (\vec{v} \cdot \vec{\nabla}B_x)\vec{e}_x + (\vec{v} \cdot \vec{\nabla}B_y)\vec{e}_y + (\vec{v} \cdot \vec{\nabla}B_z)\vec{e}_z = (v_x\frac{\delta B_x}{\delta x} + v_y\frac{\delta B_x}{\delta y} + v_z\frac{\delta B_x}{\delta z})\vec{e}_x + (v_x\frac{\delta B_y}{\delta x} + v_y\frac{\delta B_y}{\delta y} + v_z\frac{\delta B_y}{\delta z})\vec{e}_y$$

$$+ (v_x\frac{\delta B_z}{\delta x} + v_y\frac{\delta B_z}{\delta y} + v_z\frac{\delta B_z}{\delta z})\vec{e}_z = v_x\frac{\delta B_x}{\delta x}\vec{e}_x + v_x\frac{\delta B_y}{\delta x}\vec{e}_y + v_x\frac{\delta B_z}{\delta x}\vec{e}_z$$

$$= \frac{dx}{\delta t}\frac{\delta B_x}{\delta x}\vec{e}_x + \frac{dx}{dt}\frac{\delta B_y}{\delta x}\vec{e}_y + \frac{dx}{\delta t}\frac{dB_z}{\delta x}\vec{e}_z = \frac{-dB_x}{dt}\vec{e}_x + \frac{-dB_y}{dt}\vec{e}_y + \frac{-dB_z}{dt}\vec{e}_z = -\frac{\delta \mathbf{B}}{\delta t}$$

One should note that **dB/dx** has a sign that differs from that of **dB/dt**. This is why the quotient **dx / delta x** equals -1.

(58) is identical with Equation 116 in A. Föppl (1907), § 33, p. 116 (the primed differential quotient on the left-hand side of Föppl's equation must be set to zero; that is necessary since nothing changes with time in the primed rest frame of the moving charge). Moreover, (58) is a perfect match with our equation (43) above.

Because of Faraday's law, (58) can be converted into:
(59)

$$-\frac{\delta \vec{\boldsymbol{B}}}{\delta t} = \vec{\nabla} \times (\vec{\boldsymbol{B}} \times \vec{v}) = \vec{\nabla} \times \vec{\boldsymbol{E}}_{total} \Rightarrow \vec{\boldsymbol{E}}_{total} = (\vec{\boldsymbol{B}} \times \vec{v}) - \vec{\nabla}P_1$$

$\mathbf{P_1}$ **(t,x,y,z)** is an electric potential, and its gradient is an electric field. The curl of a gradient is always zero, and therefore we have:
(60)

$$\vec{\nabla} \times [(\vec{\boldsymbol{B}} \times \vec{v}) + \vec{\nabla}P_1] = \vec{\nabla} \times (\vec{\boldsymbol{B}} \times \vec{v})$$

This is why adding the gradient of $\mathbf{P_1}$ is justified in (59).

Since **B** is equal to the curl of the vector-potential **A**, we also get:
(61)

$$-\frac{\delta \vec{\boldsymbol{B}}}{\delta t} = -\frac{\delta(\vec{\nabla} \times \vec{\boldsymbol{A}})}{\delta t} = -\vec{\nabla} \times \frac{\delta \vec{A}}{\delta t} = \vec{\nabla} \times \vec{\boldsymbol{E}}_{total} \Rightarrow \vec{\boldsymbol{E}}_{total} = -\frac{\delta \vec{A}}{\delta t} - \vec{\nabla}P_2$$

$\mathbf{P_2}$ is another electric potential, whose gradient is another electric field (with zero-curl).

From (59) and from (61) follows:
(62)

$$\vec{\boldsymbol{E}}_{total} = (\vec{\boldsymbol{B}} \times \vec{v}) - \vec{\nabla}P_1 = (\vec{\boldsymbol{B}} \times \vec{v}) + \vec{\boldsymbol{E}}_{static} - \frac{\delta \vec{A}}{\delta t}$$

And also:
(63)

$$\vec{E}_{total} = -\frac{\delta\vec{A}}{\delta t} - \vec{\nabla}P_2 = -\frac{\delta\vec{A}}{\delta t} + \boldsymbol{E}_{static} + (\vec{B} \times \vec{v})$$

And thus, as the "mother of all equations" that describe the electric field of an electric charge in motion, we have:
(64)

$$[\vec{E}_{total} = (\vec{B} \times \vec{v}) - \vec{\nabla}P_1 = -\frac{\delta\vec{A}}{\delta t} - \vec{\nabla}P_2 = -\frac{\delta\vec{A}}{\delta t} + \boldsymbol{E}_{static} + (\vec{B} \times \vec{v})]$$

$$\Rightarrow \quad [\vec{\nabla} \times (-\frac{\delta\vec{A}}{\delta t} + \boldsymbol{E}_{static}) = 0 \quad \wedge \quad \vec{\nabla} \times [(\vec{B} \times \vec{v}) + \boldsymbol{E}_{static}] = 0]$$

The gradient of $\mathbf{P_1}$, which is an electric field, is composed of two partial fields: the electrostatic field $\mathbf{E_{static}}$ of the moving charge, and the electric field **-dA/dt**. These are fields which undoubtedly exist when the charge is moving. When the charge is in motion, its electrostatic field gets relativistically contracted (see above). Then the curl of the electrostatic field $\mathbf{E_{static}}$ of a charge in motion is not zero [see E.M. Purcell (1985/2011), Chapter 5.6., Fig. 5.14, p. 186: "*For in this field the line integral of E' is not zero around every closed path.*"]. The curl of the electric field **-dA/dt** (which points in a direction parallel or anti-parallel to the trajectory of the charge particle, strictly at right angle with respect to the field **B x v**) is not zero either. But the curl of the two fields combined ($\mathbf{E_{static}}$ and **-dA/dt**) *is* zero. Similarly, the gradient of $\mathbf{P_2}$ is an electric field that is also composed of partial fields, these fields being $\mathbf{E_{static}}$ and **B x v**. The curl of each of the two fields is non-zero (given the charge is in motion), but the curl of the two fields combined is zero everywhere.

When all vectors are perpendicular to each other, (64) gives for a charge moving charge that is the source of a magnetic field **B** (when replacing **B** by $\mathbf{v/c^2}$ **x** $\mathbf{E_{stat}}$ according to the Lorentz-Einstein transformation):
(65)

$$|\vec{E}_{rel}| = |\vec{B} \times \vec{v}| = |\, (\frac{\vec{v}}{c^2} \times \vec{E}_{stat}) \times \vec{v} \,| = \frac{v^2}{c^2}\boldsymbol{E}_{stat}$$

(64) and (65) solve the Trouton-Noble paradox: In the (primed) reference frame (rest frame) of the two moving charges where **v'=0**, both **-dA'/dt'** and **v' x B'** are zero, and no Lorentz-force exists. It is only the uncontracted electrostatic field $\mathbf{E'_{static}}$ of the other charge that exists. In the unprimed reference frame of the lab, the field $\mathbf{E_{static}}$ of a single charge is contracted, and both the electric fields **-dA/dt** and **B x v** (both are generated by that charge) are non-zero. But the force-effect of the electric field **B x v** on the other charge **q** is completely neutralized by the Lorentz-force **q(v x B)** on that charge. This is how the relativity principle is observed.

(65) reminds us of (42). In the 19th century, Weber and Kohlrausch found it remarkable that there was a special speed **c**, at which the electric force on a charge was as strong as the magnetic force. We can now say in what sense this is true: According to (65), the relativistic electric field of a moving charge traveling at the speed **c** is exactly as strong as the electrostatic field of that charge. Note that (42) did not give us a Lorentz force, but an electric field, just as (65) does.

One should also note that the relativistic electric field $\mathbf{E}_{rel}$=**B x v** of a moving **B**-field-source is well known from the Lorentz-transformation of electric and magnetic fields. See only R.P. Feynman (1965), Chapter 36-3, Table 26-4, where, in a primed system, the relativistic electric field of a moving magnet is given as **E'= k(v x B)**. This can be re-written as **E' = -v' x kB = -v' x B' = B' x v'**. But it is commonly applied to (electrically neutral) *magnets* (as sources of a magnet field **B**) only, not to moving *electric charges*. This restriction is unfounded, as has been shown above. In addition, it is incoherent. A moving magnet can be considered as being composed of two moving charge-sheets of different signs, with each sheet generating an effect that superposes the other.

[One should finally note: The relativistic electric field **E = B x v** has be distinguished from the Lorentz force **F=q(v x B)**, which acts on charges moving with velocity **v**. So **v** stands for the velocity of the source of the magnetic field in **E = B x v**, but for the velocity of the moving charge in **F= q(v x B)**. The distinction was stressed by A. Einstein (1905/1952), p. 37:

"*For if the magnet is in motion and the conductor at rest, there arises in the neighborhood of the magnet an electric field with a certain definite energy, producing a current at the places where parts of the conductor are situated. But if the magnet is stationary and the conductor in motion, no electric field arises in the neighborhood of the magnet. In the conductor, however, we find an electromotive force, to which in itself there is no corresponding energy,* ....".

The distinction is of uttermost importance when applying the Poynting-vector **E x B**. Only the relativistic electric field qualifies as "**E**", whereas the Lorentz force does not.]

In all pictorial renditions of the electric field of a moving charge, only the compressed field $\mathbf{E}_{static}$ is represented; authors are not aware of the other two fields [although the existence of the electric field **-dA/dt** is generally acknowledged, see only R.P. Feynman (1965), chapter 15-6, Table 15-1, where the left half of (63) can be found].

3) The role of a fourth spatial dimension in avoiding an apparent violation of Gauss's law and of the principle of charge-conservation

The relativistic electric field $\mathbf{E}_{rel}$ = **B x v** is cylindrically symmetrical with respect to the (straight) trajectory of a point-charge, and points towards the trajectory without any component in the **x**-direction. The divergence of that electric field in three-dimensional space (subscript 3D) is non-zero:
(66)

$$\vec{\nabla}_{3D} \cdot \vec{E}_{rel} = \vec{\nabla}_{3D} \cdot (\vec{B} \times \vec{v}) = (\vec{\nabla}_{3D} \times \vec{B}) \cdot \vec{v} \neq \frac{\rho}{\varepsilon_0} = 0$$

But no charge can be found at places where these field lines end or begin. The same is true for the field **-dA/dt**. That field, too, has a non-zero divergence, but no charges can be found at the end or the beginning of the straight field-lines, all of which run parallel to the **x**-axis, that is, to the straight trajectory of the visibly moving charge.

This appears to be at odds with Gauss' law. Despite the first impression, there is no violation of principles here. We have a fourth spatial dimension available. For $\mathbf{T}^{40}$ it does not matter whether the **w**-coordinate of the charge it describes is zero or non-zero. However, in all cases in which the **w**-coordinate of a charge is only a differentially small amount larger than the **w**-extension of the vast majority of charge particles in this world, it is "invisible". We thus get:
(66a)

$$\vec{\nabla}_{4D} \cdot \vec{\boldsymbol{E}}_{\boldsymbol{rel}} = \frac{\rho}{\varepsilon_0} \neq 0$$

And also:
(66b)

$$\vec{\nabla}_{4D} \cdot -\frac{\boldsymbol{d\vec{A}}}{dt} = \frac{\rho}{\varepsilon_0} \neq 0$$

This is how Gauss's law is saved. [Different from the visible charge, the invisible charge that is located in the direction of the fourth spatial dimension does not generate its own relativistic electric field in the way the visible charge does. Otherwise a runaway-effect would set in. In other word: (65) does not apply to that charge, since it cannot be ascribed a field $\mathbf{E}_{\mathbf{static}}$.]

Nevertheless, that special charge (sitting a short distance **dw** away in the fourth spatial dimension) vanishes when the moving, visible point-charge that generates the relativistic electric field and the field **-dA/dt** comes to rest. This appears to be a form of violation of the principle of charge conservation.

Again, there is no violation of principles. We have the tensor element $\mathbf{T}^{44}$. This element stands for charge-flux in the positive and negative direction of a fourth spatial dimension. The covariant divergence of the vector $\mathbf{T}^{40}$, $\mathbf{T}^{41}$, $\mathbf{T}^{42}$, $\mathbf{T}^{43}$, $\mathbf{T}^{44}$ not only throws out those apparent violations of the law of charge conservation which are mere results of spacetime curvature (if any). In addition, it does not consider the popping up or vanishing of charge as a violation of the principle of charge-conservation if what appears to be a popping-up or a vanishing is simply the result of a charge-flux from and into the fourth dimension.

In other words: The principle of conservation of charge is observed despite a vanishing or popping-up of electric charge, if any change in $\mathbf{T}^{40}$ (charge-density in Coulomb per m^3) with time is compensated by a change in $\mathbf{T}^{44}$ (charge-flux in Coulomb per m^2 and per second) with distance in the direction of the fourth dimension. The space-cell enclosing the charge is four-dimensional, and, whenever it is assumed to have the shape of a differentially small cube, it has eight sides and not just six. When using the two invisible sides of the cube (whose normals point in the direction of a fourth spatial dimension) for entering and leaving, the coming and going of charge does not violate the principle of charge-conservation.

XIII. A suggested experiment

1) First variant

The new consequence we are drawing from Kaluza's modified theory (existence of electric field lines that end or begin at places in space where no charge is visible) can be tested empirically. We imagine an infinite, flat sheet evenly charged with electricity. This sheet shall move in a tangential direction at constant speed. In this situation, the field **-dA/dt** is zero everywhere. But the electric field **B x v** is not. Consequently, the density of the total, homogeneous electric field just above the sheet increases stronger than just with $(1-v^2/c^2)^{-1/2}$. That is, it exceeds the increase that would be expected as a consequence of a relativistic length-contraction (if any). A measurement of the increase in electric field strength would thus reveal that not all of the parallel electric field lines have their starting- or end-points in electric charges that are visible.

In the (now) primed reference frame of the lab, the total electric field $\mathbf{E'_{total}}$ just above the moving sheet would be (given that **B'** in **B' x v'** can be substituted by $\mathbf{v'E'_{static}/c^2}$, and given $\mathbf{E'_{static}}$, in turn, can be replaced by $\mathbf{kE_{static}}$, with **k** being the relativistic factor of length contraction, and also given that the vectors **B'**=k**B** and **v** are at right angle with respect to each other, and finally given the equality $\mathbf{v'^2 = v^2}$):
(67)

$$E'_{total} = E'_{static} + B'v' = E'_{static} + \frac{v^2}{c^2} E'_{static} = \sqrt{\frac{1}{1-v^2/c^2}}\, E_{static} + \frac{v^2}{c^2}\sqrt{\frac{1}{1-v^2/c^2}}\, E_{static}$$

This is the postulated outcome of the experiment – against conventional wisdom according to which there is no second summand.

2) Second variant and, in case of a success, a solution of the Ehrenfest paradox

The new electric field **B x v** , which is now adapted to the special situation of our proposed experiment, even solves another problem (related to the Ehrenfest paradox): Imagine the evenly charged (thin) sheet is not flat, but forms a cylinder that rotates around its axis of symmetry. Even then, **dA/dt** is zero everywhere. Can the spinning cylinder reduce its circumference as a result of relativistic length-contraction?

The answer is in the negative, especially if we assume that the interior of the cylinder is filled with another cylinder that does not take part in the rotation. On the other hand, in the limit of a very large radius, a rotation is, over a limited distance, indistinguishable from a straight path. Even centrifugal forces then go to the limit of vanishing. However, the following difference with respect to a straight path remains: In flat Minkowski-spacetime, any spatial circle, no matter how large its radius, is not a geodesic, and an observer traveling along a spatial circle cannot consider himself or herself at rest. For after having completed a full circle, a twin has become younger than his twin-brother or twin-sister who has stayed at home, and this relationship is not reciprocal.

In order to deepen our understanding why a circle is never a geodesic, we consider a weight tight to a rope as in a hammer throw. Because of the inward (centripetal) force which the rope exerts on the weight, it is clear from the start, that is, by definition of geodesic, that the trajectory cannot be a geodesic. But things are not different when it comes to an orbit of a gravitationally bound test-mass around a spherical mass in space. According to (14), Einstein's field equation can be written as:

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu}R = -R^{\mu\nu} = kT^{\mu\nu}$$

Inside the spherical, non-rotating mass, we have:

$$R^{00} - \frac{1}{2} g^{00}R = -R^{00} = kT^{00} \neq 0$$

All the other components of $\mathbf{T}^{\mu\,nu}$ and hence of $\mathbf{R}^{\mu\,nu}$ are zero. As is commonly known, the Ricci tensor $\mathbf{R}^{\mu\,nu}$ is an expression of how the shape of a figure is deformed as one moves along a geodesic. In a four-dimensional reference frame which includes time, $\mathbf{R}^{00}$ is an expression of how the "temporal component" of the figure, that is, the local velocity of a test-object, changes with local **ds** along a geodesic. This brings us back to the cosmic variant of the Schwarzschild solution, where $\mathbf{R}^{00}$ was proportional to the squared Hubble-constant. Given a test-body on a geodesic inside the spherical mass thus cannot but steadily change its local speed with distance covered along the geodesic, there can, for symmetry reasons, only be one geodesic: the radial fall.

Back to the rotating cylinder. The circumference of the cylinder has, for the reason given above, stayed invariant when measured with meter-sticks at rest in the reference frame of the outside observer. This statement is valid in both frames of reference. The situation is analogous to an observer at rest in a gravity field, for whom clocks outside the gravity field go faster than his own clock, and for whom radially oriented meter-sticks at rest outside the gravity field are longer than his own. This relationship is absolute and not reciprocal.

However, the increase in length of circumference is not immediately realized by the co-rotating observer (but goes unnoticed), as his or her body, too, is subject to a length-dilation in the tangential direction. Only if he or she measured the length of a local, co-rotating meter-stick by means of the time needed for a light signal to travel from one end to the other, would he or she detect that the meter-stick is more than one meter long.

We thus find: The charge density of the surface of the cylinder has decreased for the co-moving observer who measures length with the help of light-signals and the local time needed to cover short local distances, but has stayed constant for the outside observer.

Let us now assume the surface of the inner cylinder is also evenly charged, but with charge-particles of the opposite sign. Let the inner cylinder also spin (in the same direction), but at an angular velocity that is somewhat different from that of the outer cylinder. We are hence facing a bar-shaped, rotating electromagnet. Neither the circumferences of the cylinders nor the density of the "visible" charge on their surfaces are affected by the rotation (for an outside observer). Nevertheless, the spinning, bar-shaped electromagnet generates the same electric field as does a spinning permanent bar-magnet. The electric field of the latter is:
(68)

$$\vec{\boldsymbol{E}}_{total}(x,y,z) = \sum_{i=1}^{i=n} \vec{\boldsymbol{B}}_i \times \vec{\boldsymbol{v}}_i$$

Each $\mathbf{B_i}$ is the magnetic field generated by a tiny volume-element of the magnetic material. There are **n** volume elements in total. The speed $\mathbf{v_i}$ is the (tangential) velocity of the respective volume element **i** of the magnetic material.

But how can it be that the spinning, bar-shaped electromagnet generates the same electric field as a spinning permanent magnet does, although both **dA/dt** and $\mathbf{E_{static}}$ are zero? (To recall: the net field $\mathbf{E_{static}}$ is zero, because rotation does not affect the charge-density of the two cylinders, so that even different speeds of motion along the circumference cannot lead to a difference in density between positive and negative charge carriers.)

The relativistic field $\mathbf{E_{rel}} = \mathbf{B} \mathbf{x} \mathbf{v}$ (generated by charge in uniform motion) comes to the rescue. It is the only electric field that remains, and it now reads:
(69)

$$\vec{E}_{total}(x,y,z) = \vec{E}_{rel} = \sum_{i=1}^{i=n} [(\frac{\vec{v}_{+_i}}{c^2} \times \vec{E}_{static_{+_i}}) \times \vec{v}_{+_i}] + \sum_{i=1}^{i=n} [(\frac{\vec{v}_{-_i}}{c^2} \times \vec{E}_{static_{-_i}}) \times \vec{v}_{-_i}] \neq 0$$

In each volume element **i**, the number of charge particles of one sign is the same as the number of charge particles of the opposite sign. Therefore the magnitude of each $\mathbf{E_{static}}$ that pertains to a single volume-element **i** is the same for the positive and the negative $\mathbf{E_{static}}$. But the velocity **v** of charge-particles in a given volume-element **i** is not the same for the two signs (it is the same in direction, but not in magnitude). This leads to a non-zero result of the summation. [Note that in (69) the speed **v** is the velocity of charge particles in a volume element, not of the volume element as such. The velocity of the volume element **i** is the average of the velocities of the positive and of the negative charge-carriers in the volume element.]

It is obvious now that textbooks which explain the relativistic electric field of a spinning, bar-shaped electromagnet by the Lorentz-contraction are wrong. R.W. Pohl (2018) is an example. His Equation 7.15 (Chapter 7.5, p. 135), which gives the magnitude of the relativistic field as "$\mathbf{E'_{rel}} = \mathbf{kvB}$", is correct. But the way R.W. Pohl obtains this result is incoherent. If the relativistic field were the result of a relativistic increase in charge-density due to length contraction (different for positive and for negative charge particles), and if we faced a special situation in which only the negative charge-carriers were in motion while the positive charge-carriers were at rest (in other words: only one of the two electrically charged, concentric cylinders is spinning while the other is not), the textbooks' equation of the relativistic electric field would consequently have to look like this (because of the alleged length-contraction of negativily charged surfaces):
(70)

$$E_{rel}^{/} = \sqrt{\frac{1}{1-v^2/c^2}}\, E_{-static} + E_{+static} = (\sqrt{\frac{1}{1-v^2/c^2}} - 1)\, E_{-static}$$

However, assuming that a length-contraction does not exist, but that a relativistic electric

field **E'$_{rel}$ = B' x v' = kB x v'** (with **v'** being the velocity of negative charge-carriers and not of the magnet as a whole) *does* exist, we have to formulate:
(71)

$$E^{/}_{rel} = \frac{v^2}{c^2}\sqrt{\frac{1}{1-v^2/c^2}}\,E_{-static}$$

The two equations (70) and (71) differ from each other. An experiment could check which of the two equations is correct in a physical sense. Since a positive empirical test of (71) would also be a confirmation that there are electric field lines which end or begin at places where no charge is "visible" (see above), a positive outcome would be proof of the existence of a fourth spatial dimension. For when presuming the validity of Gauss's law, charge must sit right there.

3) An excursion into the Kerr metric that could shed light on "dark matter"

At this point, an excursion into the Kerr-metric seems to be worthwhile: If the spherical mass is spinning, not only the tensor element **R^{00}** (= **-kT^{00}**), but also the tensor elements **R^{03}** (= **-kT^{03}**) and **R^{33}** (= **-kT^{33}**) are non-zero. This proves that the geodesic is no longer a strictly radial line: When considering, for instance, the situation in the equatorial plane, the non-zeroness of the elements **R^{03}** and **R^{33}** tell us that the **phi**-extension of a figure changes with its motion along the geodesic for a local observer at rest in the gravity field. But because of (36) and (37), this can only be explained by assuming that space is not strictly flowing in an anti-radial direction, but has a tangential component as well: Only because of that tangential component of motion of space (in which the figure is embedded) is it that the figure's tangential extension can undergo a relativistic change.

The spherical mass thus drags space along with its spinning motion. As a consequence, the orbital velocity of a test-body orbiting the spherical mass in that direction is higher than it would be without the spin of the spherical mass.

If a whole cluster of stars spinned like a rigid, spherical body, the effect would be enormous: At a given homogeneous mass density, that is, a given **R^{00}** (= **-kT^{00}**), the magnitude of the tensor elements **R^{03}** (= **-kT^{03}**) and **R^{33}** (= **-kT^{33}**) would increase proportionally with the distance to the center of the cluster. So would the tangential component of an otherwise anti-radial space-flow. A scrutiny reveals for a spinning spherical body (spherical when non-spinning) of homogenous density that has the size of a galaxy like the Milky Way, when neglecting summands that vanish when **r** is very large [see A. Trupp (2024a), Equation 61, p. 233]:

$$v_{\tan} = \frac{2GJ}{c^2 r^2}$$

With Newton's constant **G** = 6.67 x 10^{-11} m³/(kg sec²), with **c²** = 0.9 x 10^{17} m²/sec², with **r²** = 3 x 10000 lightyears x 3 x 10000 lightyears = 9 x 10^{40} m², and with the angular momentum **J** of our own galaxy, the Milky Way, being 1 x 10^{67} Joule/sec [see I. Karachentsev (1987), Chapter 7.1, between Eq. 7.2 and 7.3] , we get:

$$v_{\tan} = \frac{2GJ}{c^2r^2} = \frac{2 \cdot 6.67 \cdot 10^{-11} \cdot 10^{67}}{0.9 \cdot 10^{17} \cdot 9 \cdot 10^{40}} \; m/\sec = 0.1647 \; m/\sec$$

The result is in the right numerical order. If the equation were wrong in principle, it would be a strange coincidence to find that the quotient of numbers one of which is in the order of 10^{67} does not yield an unphysical result of, say, 10^6 m/sec, or of only 10^{-6} m/sec.

For a disc like the Milky Way, a higher $\mathbf{v_{tan}}$ can be expected than for a sphere.

As long as one is not aware of this phenomenon produced by the Kerr metric, one might be misled to infer that the mass of the spherical body has mysteriously increased by some "dark matter".

XIV. The importance of the new element T^{40} of the mass-momentum tensor

We recall that the tensor $\mathbf{T^{\mu\, nu}}$ can be thought of as being composed of four vectors, either the five horizontal lines, or the five vertical columns of the tensor. The first vertical column of the tensor is the vector (in polar coordinates):
(72)

$$\vec{T}^{\mu 0} = T^{00}\,\vec{e}_t + T^{10}\,\vec{e}_r + T^{20}\,\vec{e}_\theta + T^{30}\,\vec{e}_\varphi + T^{40}\,\vec{e}_\alpha$$

We also recall that the covariant divergence of that 5-vector is zero. We also recall that $\mathbf{T^{00}}$ was mass density (kg/m³), and $\mathbf{T^{10}}$, $\mathbf{T^{20}}$, $\mathbf{T^{30}}$ was mass flux [kg/ (m² sec)] in the three familiar spatial directions. Consequently, $\mathbf{T^{40}}$ is not only electric charge density (Coulomb/m³), but also mass flux [kg/ (m² sec)] in the fourth spatial direction.

One may regard it as a surprise or not: The two apparently distinct meanings of $\mathbf{T^{40}}$, that is, charge density in **Coulomb /m³**, but also mass flux in the direction of the fourth spatial dimension in **kg** per **m²** and per **sec**, have the same units! In basic units of **kg**, **m** and **sec**, one Ampere (**Amp**) is the current-strength through a one-meter-long section of a straight wire, which, at a distance of one meter from a parallel wire that carries the same current, experiences a Lorentz force (from the magnetic field of the parallel wire) of 2 times 10^{-7} **N** (with **N** having the dimension of **kg m / sec²**). With 1 Coulomb being 1 **Amp** times **sec** and thus 2 times 10^{-7} **N sec**, we find:
(73)

$$1\,\frac{Coulomb}{m^3} = 2 \cdot 10^{-7}\,\frac{kg}{m^2 \sec}$$

In other words: Charge-density, which the tensor element $\mathbf{T}^{40}$ stands for, has the same dimensions (units) as has mass-flux (in the direction of the fourth spatial dimension), which this tensor-element stands for as well.

We realize: *The introduction of evenly distributed electric charge in the interior of the non-spinning spherical mass leads to a situation as if the charge particles were moving in the direction of a fourth spatial dimension. More precisely: Since it does not make a difference for the* $\mathbf{T}^{\mu\ \nu}$*- tensor whether the central mass is charged with electricity or whether it is void of electric charge, but is permanently moving in the direction of the fourth spatial dimension, this cannot make any difference for the* $\mathbf{g}^{\mu\ \nu}$ *-tensor either.* (Whether the motion is in the positive or negative direction depends on the sign of the charge.)

As regards the units of the other new tensor elements, namely $\mathbf{T}^{41}$, $\mathbf{T}^{42}$, $\mathbf{T}^{43}$, $\mathbf{T}^{44}$, which are non-zero in case of a motion of charge, we similarly find:
(74)

$$1\ \frac{Coulomb}{m^2\ \sec} = 2 \cdot 10^{-7}\ \frac{kg}{m\ \sec^2} = 2 \cdot 10^{-7}\ \frac{kg\ \frac{m}{\sec}}{m^2\ \sec}$$

Charge-flux (in directions of all four dimensions of space), which these tensor-elements stand for, thus has the same dimensions (units) as have **r**-, **theta**-, **phi**- or **alpha**-components of momentum-flux, which these tensor-elements stand for as well. (The **alpha**-component of the momentum-flux we speak of is the **alpha**-component of a momentum-flux through a unit-plane whose normal points in the fourth spatial dimension, that is, in the direction of the short arc **rdalpha**. The **r**-, **theta**- and **phi**-components of momentum flux are the other components of that flux through the same unit-plane).

The beauty of all this sameness is breathtaking! It reveals a certain kind of unity of gravitation and electricity.

Since $\mathbf{T}^{40}$ does not change with time, we can in addition say: The presence of electric charge in the non-spinning spherical mass has the same influence on the metric tensor $\mathbf{g}^{\mu\ \nu}$ as would have a permanent mass-flux in the direction of the fourth spatial dimension in which mass that is departing into the fourth spatial dimension is steadily replaced by mass that is coming from the fourth spatial dimension.

This resembles the situation we found in the realm of the Kerr metric: There, too, a permanent mass-flux existed. Its direction was **phi**, and it was thus the element $\mathbf{T}^{30}$ which was non-zero inside matter. Now that role has shifted to $\mathbf{T}^{40}$. Different from the rotating spherical mass that is the starting-point of the Kerr metric, the new mass flux does not depend on **r**, but only on the charge-density.

Even more: Since $\mathbf{T}^{40}$, that is, mass-flux in the direction of a fourth spatial dimension, is non-zero as the result of the presence of stationary charge, there is good reason to assume that it could be non-zero in other ways as well. It is the gateway to the fourth spatial dimension. Hence, if some electromagnetic energy or mass suddenly appeared or disappeared, we would not be compelled to regard this as a violation of the principle of conservation of mass or energy, as long as this appearance or disappearance were in accord with Maxwell's laws.

But wait: When we dealt with the equivalence principle in the realm of the inner Schwarzschild solution, we found that it postulated a permanent flow of space-cells at the center of the spherical mass in the direction of a fourth spatial dimension (in the reference frame of an outside observer). Such a flow is *not* given an expression by $\mathbf{T^{40}}$. Space is not the same as mass or energy: We recall: The inner Schwarzschild solution is based on the 4 x 4 version of Einstein's field equation, which presupposes the conservation of mass in three-dimensional space. Any loss of mass or energy through a door to the fourth-dimension would therefore be a contradiction in itself. By contrast, the 4 x 4 version of Einstein's field equation allows a loss of space-cells.

Finally: what about the tensor element $\mathbf{T^{44}}$ (momentum-flux in the direction of a fourth spatial dimension)? Is it zero in case the non-spinning spherical body is electrically charged? The answer is: Given the element $\mathbf{T^{40}}$ is non-zero because of the presence of charge – so that the tensor $\mathbf{T^{\mu\, nu}}$ behaves exactly as it would if there were no charge, but a mass flux in the direction of the fourth spatial dimension – the element $\mathbf{T^{44}}$ cannot be zero. Otherwise the mass-flux, described by $\mathbf{T^{40}}$, would not be accompanied by a momentum-flux. But that would be incoherent.

XV. Consequences of Kaluza's modified theory for the metric tensor

1) Consequences for the element $\mathbf{g_{40}}$ of the metric tensor

Finally, let us turn our attention to the new metric tensor $\mathbf{g^{\mu\, nu}}$ or its inverse $\mathbf{g_{\mu\, nu}}$.

It is well known that, if a non-spinning spherical mass is electrically charged (with the charge being evenly distributed in the interior of the mass), the "line-element" as given by the Schwarzschild solution is modified. The Reissner-Nordström solution (found in 1918) of Einstein's field equation of a non-spinning, electrically charged spherical body modifies the (outer) Schwarzschild solution as follows (in polar coordinates):
(75)

$$d\tau^2 = \left(1 - \frac{r_s}{r} + \frac{q^2 G}{4\pi\varepsilon_0 c^4 r^2}\right) dt^2 - \frac{1}{c^2\left(1 - \frac{r_s}{r} + \frac{q^2 G}{4\pi\varepsilon_0 c^4 r^2}\right)} dr^2 - \frac{r^2}{c^2}(d\theta^2 + \sin^2\theta \; d\varphi^2)$$

Given that $\mathbf{r_s}$ (Schwarzschild radius) is an expression of the mass of the spherical body, the additional summand (in comparison with the Schwarzschild solution) that contains the electric charge **q** is nothing but an expression of the additional mass introduced by the energy of the electric field (**G** is Newton's constant). On the right-hand side of Einstein's field equation (that contains symmetrical 4 x 4 tensors), it leads to an increase in the value of the tensor-element $\mathbf{T^{00}}$, but to nothing more. As a consequence, only two diagonal elements (and no off-diagonal elements) of the metric tensor as given by the Schwarzschild solution, namely

$\mathbf{g_{00}}$ and $\mathbf{g_{11}}$, had to be modified.

The Reissner-Nordström solution does not say that it is the electric charge *as such* that co-shapes spacetime. Only the mass or energy of the electric field is supposed to do that. We shall find out whether or not both some *diagonal* elements and some *non*-diagonal elements of the metric tensor $\mathbf{g_{\mu\ nu}}$ or of its inverse (as yielded by the outer and inner Schwarzschild solutions) have to be modified other than just the way it was done by the Reissner-Nordström solution (with respect to the outer Schwarzschild solution). If so, charge as such would co-shape spacetime.

In the vicinity of a (non-spinning) spherical mass that holds evenly distributed electric charge in its interior, the "line element", based on a 5 x 5 symmetrical metric tensor $\mathbf{g_{\mu\ nu}}$, reads (when switching from polar to Cartesian coordinates; **x** is is a generalized expression of the coordinates **t**, **x**, **y**, **z**, **w**, with $\mathbf{x^0=t}$, $\mathbf{x^1=x}$, $\mathbf{x^2=y}$, $\mathbf{x^3=z}$, $\mathbf{x^4=w}$):
(76)

$$d\tau^2 = g_{\mu\nu}dx^{\mu}dx^{\nu} = a_0 dt^2 - \frac{a_1}{c^2}dx^2 - \frac{a_2}{c^2}dy^2 - \frac{a_3}{c^2}dz^2 - \frac{a_4}{c^2}dw^2 - ...$$

The coefficients $\mathbf{a_0}$, $\mathbf{a_1}$, ... $\mathbf{a_{14}}$ are the unknown 15 elements of the symmetrical 5 x 5 metric tensor $\mathbf{g_{\mu\ nu}}$. Their subscripts are numbered consecutively and do not refer to the indices of the elements of the tensor $\mathbf{g_{\mu\ nu}}$ from which they are formed. When integrating both sides of (76) (so that **delta tau** – and not **d tau** – is formed), summands that contain the differential **dw** do not take part in this operation (according to our restriction according to which differentials containing **dw** must not be integrated).

In polar coordinates ($\mathbf{x^0=t}$, $\mathbf{x^1=r}$, $\mathbf{x^2=theta}$, $\mathbf{x^3=phi}$, $\mathbf{x^4=alpha}$; under the constraint that motions in the fourth spatial dimension are restricted to differentially small distances), the line-element reads:
(77)

$$d\tau^2 = g_{\mu\nu}dX^{\mu}dX^{\nu} = a_0 dt^2 - \frac{a_1}{c^2}dr^2 - \frac{a_2}{c^2}r^2 d\theta^2 - \frac{a_3}{c^2}r^2 \sin^2\theta d\varphi^{2^2} - \frac{a_4}{c^2}r^2 d\alpha^2 - ...$$

The angle **alpha** is the angular distance from the axis of a fourth spatial dimension.

The following statement shall be proved: A (non-spinning) spherical charge *does* shape surrounding five-dimensional spacetime not only by its *mass*, but also by its *charge* as such.

In order to realize this, we recall that $\mathbf{T^{40}}$ stands for charge density. The element $\mathbf{T^{40}}$ is therefore non-zero (in units of Coulomb/m³) in the interior of the central spherical mass that carries homogenous electric charge of a single sign. We hence get from Einstein's field equation and from (14), (19):
(78)

$$[(R^{40} - \frac{1}{2} g^{40}R = k\, T^{40} \neq 0 \Leftrightarrow -\frac{1}{2} g^{40}R = 2kT^{40} \neq 0) \;\wedge\; R \neq 0] \;\Rightarrow\; g^{40} \neq 0$$

To elucidate: According to Einstein's field equation, $\mathbf{R^{40}}$ (an element of the Ricci curvature tensor), **R** (contracted Ricci tensor, that is, Ricci scalar) and $\mathbf{g^{40}}$ (element of the metric tensor) cannot be all zero, given $\mathbf{T^{40}}$ is non-zero and given **G** is a constant. The Ricci scalar **R** cannot be zero in the interior of the spherical body, since the charge does not come without mass. In addition, a spherical mass was already there when we introduced electric charge. Einstein's field equation can be re-formulated according to (14), so that $\mathbf{R^{40}}$ disappears. We then find that $\mathbf{g^{40}}$ must be non-zero according to (78) even if the spherical mass carries electric charge. And so must its inverse $\mathbf{g_{40}}$.

2) Consequences for the element $\mathbf{g_{00}}$ of the metric tensor

But that's not all of what we know about the new 5 x 5 metric tensor $\mathbf{g^{\mu\, nu}}$ or $\mathbf{g_{\mu\, nu}}$.

So far, we only know that the element $\mathbf{g^{40}}$ is affected by the introduction of charge into the non-spinning spherical mass. What can we say with regard to whether or not the element $\mathbf{g^{00}}$ (and hence its inverse) is affected (relative to what is yielded for this element by the Schwarzschild solution) in case $\mathbf{T^{40}}$ is non-zero? The answer is: As is the case in the realm of the 4 x 4 Kerr metric (which is based on $\mathbf{T^{30}}$ being non-zero), the tensor element $\mathbf{g_{00}}$ is affected, that is, modified from what it is in the inner Schwarzschild metric, also in the new 5 x 5 metric. This is because of ("**d**" refers to the modification of the Schwarzschild solution brought about by the introduction of electric charge):
(79)

$$(-\frac{1}{2} g^{00}R = 2k\, T^{00} \neq 0 \;\wedge\; dT^{00} \neq 0 \;\wedge\; R \neq 0 \;\wedge\; \frac{T^{00}}{R} \neq const) \;\Rightarrow\; dg^{00} \neq 0$$

To elucidate: Since the existence of charge in the non-spinning spherical mass has the same effect on the metric tensor as has mass-flux in the direction of the fourth spatial dimension, the kinetic energy (which has mass) of the spherical body, too, increases as it would do in case the spherical body moved in the direction of a fourth spatial dimension. So $\mathbf{T^{00}}$, which is mass density, increases. The scalar **R** in (79) is still non-zero. It depends on the mass density, which has changed. So **R**, too, has changed. However, since the change in **R** is not proportional to the change in $\mathbf{T^{00}}$, the tensor element $\mathbf{g^{00}}$ (and thus $\mathbf{g_{00}}$) must undergo a change (with respect to the inner Schwarzschild metric) as a consequence of the introduction of electric charge.

(If there are as many positive charge particles in a volume element as there are negative ones, the virtual mass flux is zero, and no virtual kinetic energy is generated. Then $\mathbf{g_{00}}$ is not affected, nor is the metric tensor $\mathbf{g^{\mu\, nu}}$ in this case.)

In the line element (76), $\mathbf{g_{00}}$ is the coefficient of $\mathbf{dt^2}$. Consequently, the quotient $\mathbf{d\, tau^2/dt^2 = g_{00}}$ is an expression of time-dilation which an observer (whose time is **tau**) at rest in the gravity field is subject to. The quotient is modified by the presence of charge in the interior of the non-spinning spherical mass. This is why charge as such, and not only the mass of its electrostatic field, co-shapes spacetime.

3) Consequences for the element g_{11} of the metric tensor

Let us now turn our attention to the element $\mathbf{g^{11}}$ of the metric tensor. For this element, we have ("**d**" refers to the modification of the Schwarzschild solution brought about by the introduction of electric charge):
(80)

$$(-\frac{1}{2}g^{11}R = 2k\,T^{11} \neq 0 \;\wedge\; dT^{11} \neq 0 \;\wedge\; R \neq 0 \;\wedge\; \frac{T^{11}}{R} \neq const) \;\Rightarrow\; dg^{11} \neq 0$$

To elucidate: The (inner) Schwarzschild solution is *not* based on $\mathbf{T^{11}}$ (momentum-flux in the direction of **r** through a unit-plane whose normal points in the **r**-direction) being zero. Instead, the momentum-flux brought about by a test-body in motion (free radial fall) has to be taken into account. This is why $\mathbf{T^{11}}$ is non-zero. The introduction of electric charge alters the momentum-flux of the test body, given $\mathbf{T^{00}}$, and thus the effective mass-density of the spherical body, has changed. Inside the spherical body, **R** is non-zero. The introduction of charge alters **R**, but not proportionally to the change in $\mathbf{T^{11}}$. Hence, $\mathbf{dg^{11}}$ is non-zero inside the spherical mass. The same is true for $\mathbf{dg_{11}}$.

4) Consequences for the element g_{44} of the metric tensor

In flat spacetime and Cartesian coordinates, four diagonal elements of the tensor $\mathbf{g_{\mu\,nu}}$ are well-known: $\mathbf{g_{00}} = 1$, $\mathbf{g_{11}} = -1$, $\mathbf{g_{22}} = -1$, $\mathbf{g_{33}} = -1$. Then we have good reason to assume that the fifth diagonal element is: $\mathbf{g_{44}} = -1$. It is non-zero, although we do not know yet how it looks like in the modified Schwarzschild metric when electric charge in added to the spherical mass.

Far away from the spherical mass, the Schwarzschild solution is indistinguishable from that for flat spacetime (see above). But when locations closer to the spherical mass or inside the mass are considered, the elements $\mathbf{g_{00}}$ and $\mathbf{g_{11}}$ of the Schwarzschild metric undergo a modification in terms of their numerical values, whereas the numerical values of $\mathbf{g_{22}}$ and $\mathbf{g_{33}}$ stay unchanged. What can be expected for the element $\mathbf{g_{44}}$ inside the spherical mass, if the spherical mass contains electric charge? The answer is: It will undergo a modification. This is because of ("**d**" refers to the modification of the inner Schwarzschild solution brought about by the introduction of electric charge):
(81)

$$(-\frac{1}{2}g^{44}R = 2k\,T^{44} \neq 0 \;\wedge\; dT^{44} \neq 0 \;\wedge\; R \neq 0 \;\wedge\; \frac{T^{44}}{R} \neq const) \;\Rightarrow\; dg^{44} \neq 0$$

As was explained above, $\mathbf{T^{44}}$ can be assumed to be non-zero inside the spherical mass as a consequence of the presence of electric charge.

XVI. Consequences for the size of the fourth spatial dimension

As regards the *size* of the fourth spatial dimension, we recall: The above equations (35), (36) and (37) describe what can be called "Einstein's ether of General Relativity". In a certain respect, this "ether" behaves like a liquid. Its physical properties, revealed by (35), (36) and (37), seem to enable relativistic effects, but also seem to make Einstein's "ether" of General Relativity a "cage" for electromagnetic fields. The latter is because electromagnetic fields seem to be confined to a thin brane of that "liquid", similar to light confined to the interior of a glass fiber. This is for purely geometrical reasons alone: If the magnitude of the electric field of a point-charge in three-dimensional space falls off with $\mathbf{1/r^2}$ and not and with $\mathbf{1/r^3}$, it is because a fourth spatial dimension either does not exist, or, if it does exist, because the electric field is confined to a brane that is extremely thin in the fourth spatial dimension. This can be inferred from the properties of a two-dimensional world. If, in that world, the field of a point-charge fell off with $\mathbf{1/r}$ and not with $\mathbf{1/r^2}$, it would be because a third spatial dimension would not exist, or, if it would exit, because the field is confined to a thin brane.

There is no reason to assume that there is no four-dimensional space outside the brane. This is because of the fact that Schwarzschild's solution leads, as is commonly known, to "Flamm's parabola" in the vicinity of a spherical mass. In L. Flamm's (1916/2015) words:

"*The whole could be regarded as some kind of a funnel surface. §4. Thus, the dimensions of the elementary rulers, which are represented by dσ, are subject to such influences in the gravitational field that using them for measurements – this is called "natural measurements of space" – does not, in general, lead to a Euclidean geometry. A quite analogous fact holds for the measurement of time by elementary clocks, the so called 'natural time measurement'* ....".

For an interpretation of this result, two alternatives are offered: The first one is to assume that Flamm's parabola does not bend into a fourth spatial dimensions, but is the result of a shortening of stationary meter sticks in three-dimensional space. The second alternative is to assume that the stationary meter sticks are not shortened, and that the parabola does indeed bend into a fourth spatial dimension. Once we have convinced ourselves that a fourth spatial dimension exists, there is no longer sound reason to believe that Flamm's parabola does NOT bend into a fourth spatial dimension macroscopically. Hence, there is no longer sound reason to believe that the fourth spatial dimension is only microscopic. On the contrary: We recall that $\mathbf{T^{44}}$ provides a door to the fourth spatial dimension for special electric charge which comes and goes with the relativistic electric field of a moving, ordinary charge.

XVII. Consequences for the number of spatial dimensions

Last not least, we realize that the number of physical dimensions is determined by the number of conservation principles: For the conservation of mass and momentum, four dimensions including time are needed. For the additional principle of conservation of charge, another spatial dimension is required. Each additional conservation principle would need an extra row and and extra column in the symmetrical $\mathbf{T}^{\mu\ \mathrm{nu}}$ tensor. That would not entail one more force is transformed into a mere phenomenon of curvature of spacetime. We remember that this was not the case for electromagnetic forces, and the only "force" which was reduced to geometry was that of gravitation..

Unless there are no additional conservation principles, no more than these five dimensions are needed.

XVIII. Results

1. Einstein's field equation is based on the (extended) relativity principle only, and on nothing else. This includes the altered version in which the cosmological constant appears as a summand. It is shown that General Relativity not only *allows* an "*interpretation*" according to which a cosmic expansion, if any, *could* be the result of "vacuum energy". Instead, the derivation of Einstein's field equation clarifies that Einstein's field equation sees "vacuum energy" *inevitably* as the *necessary* cause of a cosmic expansion. Different from the common description of that energy, Einstein's field equation requires that this energy is numerically negative (and hence "exotic") in case of a cosmic expansion (and positive in case of a cosmic contraction). Thus the apparently surprising equality of "forces" of expansion and contraction of the universe, or, in other words, the surprising proximity to its critical mass-density, seems to be a mere artefact, as it falls with the wrongly assumed positiveness of this vacuum mass or energy.

2. Einstein's field equation is an expression both of Special and of General Relativity. The relativity principle is translated into the principle of conservation of mass and momentum. This principle, in turn, is given a mathematical expression by a zero-result of the covariant divergence of all the vectors that the 4 x 4- $\mathbf{T}^{\mu\ \mathrm{nu}}$ -tensor on the right-hand side of Einstein's equation is composed of. The covariant divergence is distinguished from the ordinary divergence, as it, in accordance with the (extended) relativity principle, does not give consideration to those changes in mass and momentum that are the mere results of a curvature of spacetime. In order for Einstein's field equation not to be tautological and yielding an infinite number of solutions for one and the same constellation of masses and their motions, solutions have to be sought after for those reference frames only in which an observer feels no force on him or her. The relativity principle is thus extended (with respect to what it is in Special Relativity) to include freely falling reference frames, but must not be overstretched. The latter would be the case if literally all observers, including those who feel a force on them, were entitled to consider themselves at rest.

3. The principle of an invariance of the local speed of light is derived from Einstein's field equation, and is therefore not a basis of Special Relativity.

4. The equivalence principle is derived from Einstein's field equation, and is therefore not a basis of General Relativity. More precisely: It is derived from setting the covariant divergence of the tensor $\mathbf{T}^{\mu\ \mathrm{nu}}$ to zero. The equations then obtained lead to the conclusion that space cells are in accelerating motion from places around earth (where they emerge) towards the interior

of the earth (where they disappear). The permanent emergence and disappearance of space-cells makes it obsolete to assume (as is commonly done) that the gravitational field can be "transformed away" (by accelerating, flowing space cells) only locally in infinitesimally small regions. Instead, all gravitational fields can be a transformed away no matter how far away they are. What is commonly regarded as weight (of objects sitting on the surface) is inertia of the object's mass that refuses to "go along for the ride". Hence, heavy mass and inert mass are the same thing. Accelerating space flows are thus manifestations of the equivalence principle. The (hardly disputed) absence of a backforce-generating deformation of the electrostatic field of any charge in free fall (in a gravity field) is empirical proof of an accelerating flow of space in a gravitational field.

5. Kaluza's theory (of a unification of gravitation and electromagnetism by adding a fourth spatial dimension and thus five new tensor elements) performs perfectly when modified. The modification consists in the following: It is not the tensor elements $\mathbf{g}^{40}$, $\mathbf{g}^{41}$, $\mathbf{g}^{42}$, $\mathbf{g}^{43}$, $\mathbf{g}^{44}$ that are added as an initial step, but the elements $\mathbf{T}^{40}$, $\mathbf{T}^{41}$, $\mathbf{T}^{42}$, $\mathbf{T}^{43}$, $\mathbf{T}^{44}$. The first element is charge density, the four other elements are charge-flux in the **x**-, **y**-, **z**- and **w**- direction. The zero covariant divergence of the vector $\mathbf{T}^{40}$, $\mathbf{T}^{41}$, $\mathbf{T}^{42}$, $\mathbf{T}^{43}$, $\mathbf{T}^{44}$ then is an expression of the principle of conservation of charge. Three-and-a-half of Maxwell's four equations of electromagnetism can be derived from Einstein's field equation, if all tensors are the usual 4 x 4-tensors. The missing one-and-a-half equations can be derived from Einstein's field equation after adding the five new elements $\mathbf{T}^{40}$, $\mathbf{T}^{41}$, $\mathbf{T}^{42}$, $\mathbf{T}^{43}$, $\mathbf{T}^{44}$. Despite an interrelation between gravity and electricity, electric force on a charge cannot be "transformed away", whereas the gravitational "force" can be "transformed away" ubiquitously and not only infinitesimally. The expansion of the 4 x 4 tensors into 5 x 5 tensors in Einstein's field equation has been overdue. This is because the covariant divergence of the tensor $\mathbf{T}^{\mu\ nu}$ on the right-hand side of Einstein's field equation is an expression of the conservation principles we know of. Then, however, the principle of conservation of charge must not be missing.

6. After an expansion of all tensors in Einstein's field equation from symmetrical 4 x 4 tensors to symmetrical 5 x 5 tensors, the Trouton-Noble-paradox can now be solved to full extent. This is done by the recognition that a moving, unaccelerated electric point-charge on a straight path generates not only a magnetic field, but also a relativistic, cylindrically symmetrical, strictly radial electric field **E=B x v** around the straight trajectory of the moving charge. That electric field is thus directed perpendicular to the magnetic field and also perpendicular to the path of motion of the charge. Both the ordinary and the covariant divergence of that field would be non-zero if only three spatial dimensions existed, but no charge is visible at places where these field lines end or begin. A violation of Gauss' law is avoided by the fact that there are four – and not just three – spatial dimensions in which the divergence of the electric field is determined. The charges can only sit in the direction of the fourth spatial dimension **w** at a distance **dw** from the end of a field line. Although these charges come and go, the principle of conservation of charge is not violated. Because of the new tensor element $\mathbf{T}^{44}$, charge comes and goes through a plane whose normal points in the direction of a fourth spatial dimension. A cube in four spatial dimensions has not only six, but has eight sides.

7. An experiment similar to that conducted by H. A. Rowland in 1878 could be performed that could confirm the existence of the special relativistic electric field, and could thus, on the

basis of the validity of Gauss's law, confirm the existence of a fourth spatial dimension.

8. Although a solution of Einstein's expanded solution (that now has symmetrical 5 x 5 tensors), that is, the equation of the "line element", is still to be found for a non-spinning spherical mass with electric charge, we nevertheless find that the addition of electric charge to a non-spinning spherical mass leads to a modification of the Schwarzschild- and also of the Reissner-Nordström-metric (each in polar coordinates). The element $\mathbf{g_{40}}$ of the metric tensor turns out to be non-zero. Moreover, the tensor elements $\mathbf{g_{00}}$ and $\mathbf{g_{11}}$ are modified both with respect to the inner Schwarzschild solution and also with respect to the Reissner-Nordström solution. Astonishingly, the units of $\mathbf{T^{40}}$ are expressions both of charge-density and of mass-flux in the direction of a fourth spatial dimension, as they are (in basic units of **kg**, **m** and **sec**): 1 Coulomb/$\mathbf{m^3}$ = 2 times 10^{-7} **kg**/($\mathbf{m^2}$ **sec**). *The introduction of evenly distributed, stationary electric charge in the interior of a non-spinning spherical mass affects the* $\mathbf{T^{\mu\ nu}}$ *tensor and hence also the* $\mathbf{g^{\mu\ nu}}$ *tensor in a way just as if some mass moved in the direction of a fourth spatial dimension*! In other words: Math shows it does not matter for the tensor $\mathbf{T^{\mu\ nu}}$ whether the spherical, non-spinning mass is carrying electric charge or whether that mass, void of electric charge, is permanently moving in the direction of a fourth spatial dimension, instead. Then this distinction does not matter for the metric tensor $\mathbf{g^{\mu\ nu}}$ either. Charge as such, and not only the mass of its electric field, thus co-shapes spacetime by its virtual kinetic energy (equivalent to mass) of motion in the direction of the fourth spatial dimension.

9. Given all this, there is no longer any sound reason to believe that L. Flamm's "parabola" does not bend into a fourth spatial dimension *macroscopically*, even though the *accessible* universe seems to be pancake-like ("brane") with only a microscopical extension in the fourth spatial dimension. The "material" of that "brane" is Einstein's "ether" of General Relativity, whose properties is given a mathematical expression. According to these equations, space-cells can be in motion (either accelerating or uniform), although the velocity is frame-dependent. In the realm of the Kerr metric around spinning spherical masses, this leads to tangential components of motions of space-cells. A spinning mass thus drags space along with its spinning motion, and increases the velocity of an orbiting test mass. This leads to the wrong impression of an additional, "dark" mass.

10. The number of physical dimensions including time is determined by the number of conservation principles. For the principle of conservation of mass and momentum, four dimensions including time are needed. For the principle of conservation of charge, the existence of one more spatial dimension is indispensable.

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